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An interpolation-type orthotropic yield function and its application under biaxial tension

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A B S T R A C T

Most previous work in anisotropic yield criteria has focused on describing the yield loci using a continuous function, which is ideal when applying to materials with limited anisotropy. But in practice, many widely used metals such as aluminum alloys and magnesium alloys show strong features in different crystallographic directions, which cause increasing undetermined coefficients and more complex functional forms. Furthermore, with development of new materials, existing yield criteria shall meet challenges in feasibility because a continuous function is expected to predict only a few kinds of yield surface shapes. To extend the anisotropic yield criteria family, an interpolation-type orthotropic yield function for plane stress is proposed in this paper, in which the physical meaning of coefficients is directly defined. The proposed interpolation-type orthotropic yield function reproduces with great accuracy the anisotropic behaviors under biaxial tension and uniaxial tension/compression, including but not limited to the varying yield stress and *R*-coefficients with loading direction and tension/compression asymmetry. Moreover, comparing the predictions of the yield loci by the interpolation-type orthotropic yield function with those by classical yield functions, it is shown that the proposed function gives the best agreement with experimental data of FCC, BCC and HCP sheet metal samples.

1. Introduction

Since Hill's fundamental work in 1948 [1], many types of anisotropic yield functions have been proposed to describe the behavior of sheet metals. Hill48 model has been implemented into several FE element codes because of its user-friendly formulation. However, since the anisotropy under biaxial tension is not considered, Hill48 has been found to overestimate the biaxial yield stress for materials like steels ($R>1$), while underestimate the biaxial yield stress for materials like aluminum alloys ($R<1$). To improve the simulation performance, Hill79 [2], Hill90 [3], and Hill93 [4] models have been published subsequently, each being a modification based on Hill48. Barlat and Lian [5] proposed Yld89 by extending Hosford's [6] non-quadratic isotropic function to describe in-plane anisotropy, which provides a better predictability than Hill48 in modeling the earing. Afterwards, a series of yield functions known as Yld91 [7], Yld94 [8], Yld96 [9], Yld2000-2D [10] and Yld2004 [11] have been developed by Barlat and his coworkers. In addition to the work of Barlat, Banabic et al. proposed BBC2000 model [12] based on Yld89. Further modifications of BBC2000 were found in BBC2002 [13], BBC2005 [14,15] and BBC2008 [16]. Compared with Hill's yield criteria, the Barlat series and BBC series performed better in predicting the anisotropic coefficients and biaxial flow stress, which is at cost of increasing the complexity of the yield function. Furthermore, since all the criteria mentioned above are axisymmetric in shape, they are not suitable in describing the tension/compression flow stress asymmetry occurred in hexagonal close packed (HCP) metals such as magnesium alloys [17] and titanium alloys [18]. Concerning this issue, the Cazacu series have been proposed [19,20].

Despite the difference in equation form, all the above mentioned yield criteria are continuous functions, the undermined parameters of which are in accordance with the number of anisotropic characteristics involved [21]. Naturally, to further improve the prediction accuracy, the function is bound to be more complex. This tendency could be seen by comparisons both within each yield function series and between different series. However, with more materials with strong anisotropy being developed, continuous yield functions would be too complex for coefficient determination and numerical computation. Vegter and van den Boogaard proposed an interpolation-type yield function based on the second order Bézier curves [22]. It is demonstrated that the interpolation method is promising in accurately modeling the mechanical behavior of highly anisotropic sheet metals.

In this paper, an interpolation-type orthotropic yield criterion for plane stress has been proposed. The Hermite interpolation, involving both the function value and its first order derivative in a simple form, is used as interpolation function to describe the anisotropic yield behavior in the quasi-π plane. In this way, the anisotropy shown in

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tension and compression along any crystallographic axes could be taken into account without any increase in function complexity.

2. The interpolation-type orthotropic yield function

2.1. Basic concepts and assumptions

As an initial step of the research, the yield function postulated in this paper is limited to the condition in which the principal stresses act along the anisotropic axes and no shear stress component in the anisotropic coordinate system exists. In framing the criterion, general propositions used in the classical constitutive theory are kept. Specifically, assumptions adopted are as follows:

1) The yield surface and its normal vector are varying continuously. Therefore it is reasonable to approximate the yield locus using an interpolation method, and prediction accuracy could be improved by increasing interpolation points.
2) Yielding behavior is fully determined by the deviatoric stress state, and the effect of hydrostatic stress on yielding is neglected.
3) The plastic strain increment vector is normal to the yield surface.

2.2. Stress and strain on the quasi-π plane

2.2.1. Yield stress on the quasi-π plane

Similar to the classical π plane, a quasi-π plane has been adopted, in which each axis represents the deviatoric stress, as schematically shown in Fig. 1, to demonstrate the deviatoric stress state at yielding. For the sheet metal, the deviatoric stress in rolling, transverse and normal direction, are denoted by the component along 1′, 2′ and 3′ axes or σ'_x, σ'_y, and σ'_z in the quasi-π plane. In plane stress condition, for a random yielding point p_i (the open star symbol in Fig. 1), the transformation matrix between the direct stress vector $[\sigma_x, \sigma_y]$ and the deviatoric stress vector $[\sigma'_x, \sigma'_y, \sigma'_z]$ is expressed as follows:

$$\begin{pmatrix} \sigma'_x \\ \sigma'_y \\ \sigma'_z \end{pmatrix} = \frac{1}{3} \times \begin{pmatrix} 2, -1 \\ -1, 2 \\ -1, -1 \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \end{pmatrix} \tag{1}$$

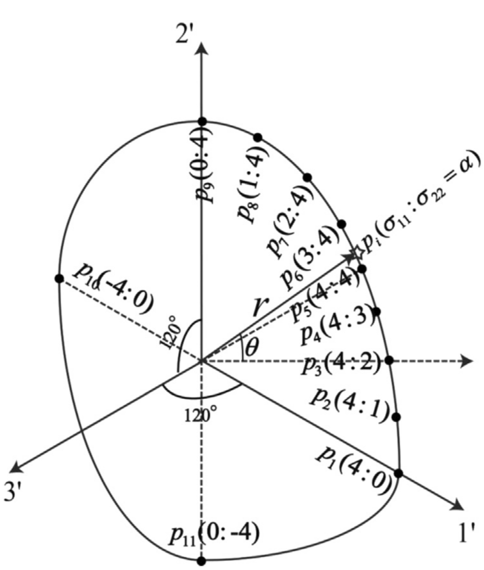

Fig. 1. Schematic yield locus on quasi-π plane.

The magnitude of the deviatoric stress vector, denoted as radius r on the quasi-π plane, is related to σ'_x, σ'_yandσ'_z by projecting r to the three axes in the relationship as follows:

$$\begin{cases} \sigma'_x = r\cdot \ \cos\left(\theta + \frac{\pi}{6}\right) \\ \sigma'_y = r\cdot \ \cos\left(\frac{\pi}{2} - \theta\right) \\ \sigma'_z = r\cdot \ \cos\left(\theta + \frac{5\pi}{6}\right) \end{cases} \tag{2}$$

The relationship between r and σ'can be rewritten as

$$r = \sqrt{\frac{2}{3}} \cdot \sqrt{\sigma_x'^2 + \sigma_y'^2 + \sigma_z'^2} \tag{3}$$

It is noteworthy that the expression in Eq. (3) is slightly different from that in the well-acknowledged π plane, where the radius $r_\pi = \sqrt{\sigma_x'^2 + \sigma_y'^2 + \sigma_z'^2}$. So for distinction, the modified π plane presented in this work is called quasi-π plane.

The angle of r with respect to the horizontal direction is calculated as follows:

$$\tan\ \theta = \frac{2\sigma_y - \sigma_x}{\sqrt{3}\sigma_x} \tag{4}$$

Assuming the ratio of $\sigma_x : \sigma_y = \alpha$, Eqs. (3) and (4) are rewritten as follows:

$$r = \frac{2}{3} \cdot \sqrt{\alpha^2 - \alpha + 1} \cdot \sigma_y \tag{5}$$

$$\tan\ \theta = \frac{2 - \alpha}{\sqrt{3}\alpha} \tag{6}$$

Eqs. (5) and (6) give the definitions of radius and angle of the yield locus on the quasi-π plane (shown in Fig. 1), both of which are determined by the stress state at yielding. Specifically, under conditions of uniaxial and biaxial tests with $\sigma_x : \sigma_y = 4:0, 4:1, 4:2, 4:3, 4:4, 3:4, 2:4, 1:4, 0:4, -4:0$ and $0:-4$, the radii and angles, calculated by Eqs. (5) and (6), are presented schematically in Fig. 1 by black solid circles numbered as $p_1 - p_{11}$, respectively.

2.2.2. Direction of plastic strain rate on the quasi-π plane

In the classical constitutive theory, the plastic strain rate space and the stress space are superposed, and their deviatoric components are also superposed on the π plane. When the material is isotropic, yield locus on the quasi-π plane is circular and the direction of plastic strain increment is in accordance with the radial direction. However, in the case of anisotropic materials, the radius is bound to be varied with directions and the direction of plastic strain rate, determined by the normal direction to the yield locus, is no longer parallel with the radial direction.

The state of the plastic strain rate of a random yield point i on the principal plane and quasi-π plane is schematically illustrated in Fig. 2. On the quasi-π plane shown in Fig. 2(b), the angle of $\vec{oi}$ with respect to the horizontal direction, i.e. the polar angle, is θ. And ω is the angle between the radial direction $\vec{oq}$ and the normal direction $\vec{op}$ of the yield locus. Point $i''$is another yield point in the vicinity of point i. The angle between $\vec{oi}$ and $\vec{oi''}$ is $\Delta\theta$. Point i' is the intersection of arc ii' and $\vec{oi''}$. When $\Delta\theta$ is close to zero, it can be shown that $\Delta ii'i''$ and Δiqp are similar triangles. It leads to

$$\frac{\Delta r}{r\Delta\theta} = \frac{\left|\vec{iq}\right|}{\left|\vec{pq}\right|} \tag{7a}$$

$$\frac{dr}{d\theta} = \lim_{\Delta\theta \to 0} \frac{\Delta r}{\Delta\theta} = r \cdot \frac{\left|\vec{pq}\right|}{\left|\vec{iq}\right|} = r\cdot\ \tan\ \omega \tag{7b}$$