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Control of geometrically nonlinear vibrations of functionally graded magneto-electro-elastic plates

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ABSTRACT

This article deals with the analysis of active damping of geometrically nonlinear vibrations of functionally graded magneto-electro-elastic (FGMEE) plates integrated with the patches of the active constrained layer damping (ACLD) treatment. The constraining layer of the ACLD treatment is composed of the vertically/obliquely reinforced 1–3 piezoelectric composite (PZC). The constrained viscoelastic layer of the ACLD treatment is modeled by using a Golla–Hughes–McTavish (GHM) method in time domain. The material properties of the FGMEE plate are assumed to be functionally graded along the thickness direction according to a simple power-law distribution. Based on the layer-wise shear deformation theory, a three-dimensional finite element (FE) model of the overall smart FGMEE plate has been developed taking into account the effects of coupling between elasticity, electric and magnetic fields, while the von Kármán type nonlinear strain displacement relations are used for incorporating the geometric nonlinearity. Influence of the variation of the power law index, material gradation, edge boundary conditions and the piezoelectric fiber orientation angle in the 1–3 PZC constraining layer of the ACLD treatment on the control of geometrically nonlinear vibrations of the FGMEE plates have been investigated.

1. Introduction

Active control of composite structures using piezoelectric sensors and actuators has received significant attention of the researchers in recent decades due to their wide range of applications in aerospace, automotive, civil, marine and medical engineering. The structures integrated with piezoelectric materials possess a self-sensing and self-controlling capabilities and are commonly called as smart structures or intelligent structures [1–3]. The initial experimental investigation on the masterly use of low-control-authority monolithic piezoelectric materials for active control of vibrations of the plates led to the development of active constrained layer damping (**ACLD)** treatment [4,5]. This research has motivated many researchers to carry out further investigations on the use of the **ACLD** treatment for active damping of composite structures. Ray et al. [6] experimentally and theoretically analyzed the **ACLD** of cylindrical shell. Chantalakhana and Stanway [7] investigated the performance of the **ACLD** treatment for clamped–clamped plate. Lim et al. [8] developed the closed loop finite element (**FE**) modeling of the **ACLD** for time domain analysis. Ray and his co-researchers [9–12] have been performing the widespread research on the performance of the **ACLD** treatment for active damping of linear and nonlinear vibrations of smart structures and they established that the damping characteristics of these structures can be enhanced significantly by using 1–3 piezoelectric composites (**PZC**) as the materials of the constraining layer of the **ACLD** treatment. Layered/laminated composite structures can be tailored to design advanced structures while the discontinuity or mismatch in the properties of each layer at the interface between the two adjacent layers may cause interlaminar shear stresses that may lead to the initiation of imperfection like delamination, crack etc. To mitigate such disadvantages a new class of advanced materials known as functionally graded materials (**FGM**) have been emerged. In an endeavor to develop the super heat resistant materials, Koizumi [13] first proposed the concept of this **FGM. FGM** materials are characterized by a smooth and continuous variation of material properties particularly along the thickness direction. A great deal of research has been reported on the composites made of **FGM**s [14–17].

Recently, magneto-electro-elastic (**MEE**) or multiferroic composites have received more attention of the researchers due to their interesting properties such as electro-elastic, magneto-elastic and electromagnetic coupling effects. In addition, these composites have the ability to convert energy from one form into the

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other (among magnetic, electric and mechanical energies). These interesting properties of multiferroic composites can be utilized for the various smart structural applications, such as, smart sensors and transducers, spintronics, optoelectronic devices, ultrasonic imaging devices, sonar applications etc. Exact solution of simply supported multilayered magneto-electro-elastic plates under surface and internal loads by using modified Stroh formalism and a propagator matrix method is demonstrated by Pan [18]. The same approach was extended to free vibration analysis by Pan and Heyliger [19]. Buchanan [20] computed the natural frequencies of vibration for **MEE** layered infinite plate and compared with multiphase composite plates. The **FE** model based on a higher order shear deformation theory for static and free vibration analysis of **MEE** plates has been developed by Moita et al. [21]. Furthermore, several methods have been implemented to investigate the free vibration analysis of non-homogenous or functionally graded magneto-electro-elastic (**FGMEE**) plates/shells namely, independent state equations by Chen et al. [22], **FE** method by Bhangale and Ganesan [23] and discrete layer method by Ramirez et al. [24]. Bishay [25] analyzed the **FGMEE** composites using node-wise material properties. Although, **MEE** structures have gained remarkable attention of the researchers in recent years, very few research on the large deflection analysis of the **MEE** plates have been reported in the literature. Sladek et al. [26] presented the meshless local Petrov–Galerkin method to analyze the large deformation of the **MEE** thick plates. Xue et al. [27] proposed the analytical solutions for the large-deflection model of rectangular **MEE** thin plates. Milazzo [28] analyzed the large deflection of **MEE** laminated plates using first order shear deformation theory and von Kármán stress function approach. Most recently, Alaimo et al. [29] proposed an equivalent single-layer model for the large deflection analysis of multilayered **MEE** laminates by the **FE** method.

Owing to the flexibility of composite structures and small material damping, vibrations induced in the structure may lead to large amplitudes. Thus, the effect of geometrically nonlinear deformations becomes prominent in the behavior of composite structures [30]. Many researchers performed the active control of geometrically nonlinear analysis of composite structures using piezoelectric sensors and actuators for attenuating the undesired vibrations. For example, Birman [31] proposed a theory of geometrically nonlinear composite plates with piezoelectric stiffeners. Pai et al. [32] studied the refined nonlinear model of piezoelectric plate laminates. Fakhari and Ohadi [33] investigated the large amplitude vibration control of smart laminated composite plate and the functionally graded material plates under thermal gradient and transverse mechanical loads using integrated piezoelectric sensor/actuator layers. Zhang and Schmidt [34,35] developed an electro-mechanically coupled geometrically nonlinear **FE** model based on large rotation shell theory for static and dynamic analysis of piezoelectric integrated thin-walled structures. They used first-order shear deformation (FOSD) hypothesis and considered six kinematic parameters expressed by five nodal degrees of freedom (DOFs) for implementing the large rotation theory. Most recently, Kattimani and Ray [36,37] studied the active damping of geometrically nonlinear vibrations of MEE plates and doubly curved shells using 1–3 piezoelectric composites. In the present study, the effectiveness of the vertically/obliquely reinforced 1–3 **PZC**s as the materials of the constraining layer of the **ACLD** treatment for active damping of geometrically nonlinear vibrations of the **FGMEE** plates has been investigated. For such investigation, three dimensional analysis of the **ACLD** of geometrically nonlinear vibrations of **FGMEE** plates integrated with the patches of the **ACLD** treatment has been carried out by the **FE** method taking into account the effects of coupling between elasticity, electric and magnetic fields. The **FE** model derived here can also be used for studying the purely elastic, piezoelectric and piezomagnetic laminated composite plates by replacing the layers of the **FGMEE** plate with the conventional composite layers, piezoelectric layers and piezomagnetic layers, respectively. However, the present investigation is devoted to the **FGMEE** plates only. The effects of various parameters such as the variation of power law index, type of gradation, the edge boundary conditions and the variation of the piezoelectric fiber orientation angle in the 1–3 **PZC** constraining layer on the control of geometrically nonlinear vibrations of the **FGMEE** plates have been investigated. It may be mentioned here that the work on the active control of geometrically nonlinear vibrations of **FGMEE** plates is not yet available in the open literature. Also, the implementation of the Golla–Hughes–McTavish (**GHM**) method for modeling the viscoelastic layer is a novel approach for deriving the **FE** model of **ACLD** of geometrically nonlinear vibrations of **FGMEE** plate.

2. Problem description and governing equations

A **FGMEE** plate integrated with a patch of the **ACLD** treatment on the top surface of the plate is schematically illustrated in Fig. 1(a) and (b). The length, the width and the total thickness of the **FGMEE** plate are $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{H}$, respectively. The thickness of the constraining piezoelectric layer and the constrained viscoelastic layer of the **ACLD** treatment are $\boldsymbol{h_p}$ and $\boldsymbol{h_v}$, respectively. The substrate of this smart **FGMEE** plate consists of three layers of equal thickness. The top and the bottom layers are **FG** while the middle layer is homogeneous with stacking sequence **FG/F/FG** and **FG/B/FG** indicating the top/middle/bottom layer, in which **B** stands for the piezoelectric material like $\mathbf{BaTiO_3}$, **F** stands for the magnetostrictive material such as $\mathbf{CoFe_2O_4}$ and **FG** represents functionally graded layer (volume fraction gradation of **B** and **F)** as shown in Fig. 1(a). The constrained viscoelastic layer is sandwiched between the host **FGMEE** substrate and the constraining layer of the **ACLD** treatment. The constraining layer of the **ACLD** treatment is made of the vertically/obliquely reinforced 1–3 **PZC** material. A layer of the obliquely reinforced 1–3 **PZC** material is illustrated in Fig. 1(c) wherein the piezoelectric fibers are coplanar with the $\boldsymbol{xz}$ plane while their orientation angle with the $\boldsymbol{z}$-axis is λ. Although not shown here, the piezoelectric fibers can be coplanar with the $\boldsymbol{yz}$-plane and their orientation angle with the $\boldsymbol{z}$-axis is also denoted by λ. In case of the obliquely reinforced 1–3 **PZC**, the orientation angle (λ) is nonzero while it is zero for the vertically reinforced 1–3 **PZC.** Fig. 2(a) and (b) illustrate the schematic representation of the kinematics of deformations of the undeformed transverse normal in the $\boldsymbol{xz}$- and the $\boldsymbol{yz}$-planes, respectively. As shown in these figures the axial displacements of any point on the mid-plane of the substrate along $\boldsymbol{x}$- and $\boldsymbol{y}$-directions are $\mathbf{u_0}$ and $\mathbf{v_0}$, respectively. The rotations of the portions of the normal lying in the substrate plate, the viscoelastic layer and the piezoelectric layer are represented by θ_x, ϕ_x and γ_x, respectively in the $\boldsymbol{xz}$-plane, while θ_y, ϕ_y and γ_y represent the same in the $\boldsymbol{yz}$-plane. Accordingly, the axial displacements **u** and **v** of any point in the overall plate along the $\boldsymbol{x}$- and $\boldsymbol{y}$-directions, respectively, can be written as

$$\mathbf{u}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{t}) = \mathbf{u}_0(\mathbf{x},\mathbf{y},\mathbf{t}) + \left((\mathbf{z} - \langle \mathbf{z} - \frac{\mathbf{h}}{2}\rangle)\right)\theta_x(\mathbf{x},\mathbf{y},\mathbf{t}) + \left(\langle \mathbf{z} - \frac{\mathbf{h}}{2}\rangle - \langle \mathbf{z} - \mathbf{h}_{N+2}\rangle\right)\phi_{\mathbf{x}}(\mathbf{x},\mathbf{y},\mathbf{t}) + \langle \mathbf{z} - \mathbf{h}_{\mathbf{N}+2}\rangle \gamma_{\mathbf{x}}(\mathbf{x},\mathbf{y},\mathbf{t}) \tag{1}$$

$$\mathbf{v}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{t}) = \mathbf{v}_0(\mathbf{x},\mathbf{y},\mathbf{t}) + \left(\mathbf{z} - \langle \mathbf{z} - \frac{\mathbf{h}}{2}\rangle\right)\theta_{\mathbf{y}}(\mathbf{x},\mathbf{y},\mathbf{t})$$