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Theoretical model for a ductile porous projectile striking on an elastic target bar

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ABSTRACT

As an extension of the Taylor model, in 2001 Lu et al. [13] made a study on the impact behavior of a flat-nosed cylindrical porous projectile in order to determine the dynamic yield stress of porous materials at a high strain rate. They assumed that the density of the compressed porous material was a linear function of the compressive strain, which was only suitable for small strain cases. In 2010, as another extension of the Taylor model, Yang et al. [16] proposed a theoretical model to study the flat-nosed solid cylindrical projectile impinging against a semi-infinite elastic bar along its axial direction, which provided a useful guide for the application of the output bar in a Hopkinson test system to obtain instantaneous deformation characteristics of the projectile. However, this investigation was only concerned with metal projectiles, where the density is a constant. The present paper is developed from both Lu's and Yang's models and an analytical model is proposed to study a ductile porous projectile impinging against a semi-infinite elastic bar. The compressibility of the porous projectile is incorporated and a nonlinear relationship is used between the density of the compressed porous projectile and strain. The plastic Poisson's ratio of the projectile can be either a constant or a function of the compressive plastic strain. For the case of constant value of plastic Poisson's ratio, the present model based on the first order Taylor series expression of the density ratio can be degenerated to the existing models. For the case that the plastic Poisson's ratio changes with the compressive plastic strain, attention is paid to the influence on the impact responses of the porous projectile, of the geometry and material properties of both the porous projectile and the target bar, and the initial impact velocity of the porous projectile. The results show that the non-dimensional material, geometry and initial velocity parameters have great effects on the compressive plastic strain and lengths of the deformed and undeformed sections of the projectile, the duration of impact-contact process and the energy partitioning between the projectile and the target bar. The present investigation provides a theoretical model for testing ductile porous materials.

1. Introduction

Due to their light-weight and high energy absorption ability, porous materials such as foams and honeycombs are widely used in the fields of aeronautics, astronautics, automobiles, and human protection. Their mechanical behavior at high strain-rates is especially important for the design of energy absorption structures. Taylor test is one of the important procedures to determine the dynamic yield stress of materials. In a Taylor test, a flat-nosed cylindrical projectile is fired to impinge against a rigid anvil, the dynamic yield strength of materials can thus be evaluated by the measurement of the mushrooming of flat-ended projectiles and the theoretical analysis.

A simple one-dimensional model of such a problem was first proposed by Taylor [1] and Whiffin [2] in 1948, to provide an estimate of the dynamic yield stress of materials. Consequently, many attempts have been made to improve the one-dimensional Taylor model. For example, Hawkyard [3] utilized an energy balance equation rather than the momentum equilibrium across the plastic wave front. Lee and Tupper [4] incorporated the effect of strain hardening on the final profile of the projectile. Ting [5] studied the impact of a viscoplastic rod on a rigid wall. Jones et al. [6] analyzed the mass loss from the rigid rod to the plastic zone and proposed a '$\alpha\beta$ model' to predict the flow stress of the material. Eakins and Thadhani [7] established a one-dimensional analytical model for the reverse Taylor anvil-on-rod impact experiment. Besides the one-dimensional models,

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three-dimensional simulations of the Taylor test were performed with the aid of modern computer codes and some new interpretations of experimental observations were obtained. For instance, with the complex three-dimensional finite element analysis, the entire plastic deformations in the Taylor test based on the energy and momentum approaches were compared [8]. Not only the deformations but also the phenomena and mechanisms of fracture in the Taylor test were revealed [9,10].

Apart from metallic materials, the Taylor test has been extended to assess the dynamic mechanical behavior of other materials such as polymers [11] and foams [12–14]. Lopatnikov et al. [12] conducted experiments on the impact of closed-cell aluminum foam cylinders onto the Hopkinson bar. A one-dimensional analytical model without considering the cross-sectional area change of the projectile and the elasticity of the impact wall was established. Based on the analytical model and the measured un-collapse length of the foam cylinder, the critical stress of the elastic, perfectly plastic rigid foam was evaluated. Especially, Lu et al. [13,14] extended the Taylor model to a compressible porous projectile striking on a rigid surface and reported their experimental investigations into the dynamic yield stresses of porous iron and bronze samples. In Lu's model [13], a very important feature of porous materials which distinguishes from that of traditional incompressible metals was captured. They considered the fact that the density of a porous material increases with the compressive plastic strain. For the sake of simplicity, they assumed that for porous materials with a given initial density, the density of the compressed projectile was a linear function of the compressive plastic strain, which actually indicated that the plastic Poisson's ratio was treated as a constant in their work. However, it is noted that the plastic Poisson's ratio of porous materials in fact varies with the compressive plastic strain. The assumption of the density of a compressed porous material as a linear function of the compressive plastic strain holds only for small compressive plastic strains [15]. Furthermore, in some test setup, a rigid anvil may not be appropriately assumed. After impact, the anvil experiences elastic deformations and elastic waves propagate in the anvil [12,16]. In Yang's model [16], an incompressible metal projectile impinging against a semi-infinite elastic bar was studied and the elastic effect of the target bar was found to play an important role in the prediction of the plastic deformation of the projectile. For compressible porous materials, there is still a lack of an analytical model that incorporates both the effect of the elastic bar and the compressibility of the porous projectile.

This paper aims at developing an analytical model for a compressible ductile porous projectile impinging normally against a semi-infinite elastic bar, which is an extension of Yang's model [16] and Lu's model [13]. The plastic Poisson's ratio of the porous projectile can be a constant or a function of the compressive plastic strain. A more complex expression rather than a linear function is given between the density of the compressed porous materials and strain. In the case of a constant plastic Poisson's ratio, the present model can be degenerated to Yang's model and Lu's model. The present model is useful for the prediction of the dynamic yield stress of ductile porous materials and the design application of ductile porous materials in light-weight and energy absorption structures.

2. Analytical model and basic equations

Consider a rigid, perfectly plastic cylindrical ductile porous projectile of length L_0, cross-sectional area A_0, material density ρ_0 and yield stress σ_Y, striking perpendicularly on a semi-infinite elastic target bar with an initial velocity U, as shown in Fig. 1a. As in Lu's model [13], the pore size of the projectile is assumed to be much smaller than the diameter of the projectile and thus the ductile porous projectile is modeled as a continuum. The cross-sectional area, elastic modulus and density of the elastic target bar are A_1, and E_1, and ρ_1, respectively. As the projectile impinges against the elastic target bar, a layer in the vicinity of the impacted surface of the projectile will yield immediately and a steep-fronted plastic wave will travel from the impact end into the projectile with velocity v, while an elastic wave travels towards the rear end of the target bar with velocity c_1. Similar to Taylor's assumption, the stress in the plastic deforming layer of the projectile is equal to σ_Y, and the strain in the undeformed section of the projectile, which is ahead of the wave front, equals to zero. The cross-sectional area and the density across the wave front change from A_0–A, and ρ_0–ρ, respectively. At any given time during an intermediate stage of impact-contact process, denote u the velocity of the undeformed section of the projectile, u_1 the common velocity of the plastic deforming layer of the projectile and the impacted surface of the target bar, z and h the lengths of the undeformed and deformed sections of the projectile, respectively, as illustrated in Fig. 1b. At the end of impact-contact process, the residual velocity of the projectile is u_{1f} and the elastic wave has traveled a distance l_f in the target bar. The final contact area and lengths of the undeformed and deformed sections of the projectile become A_f, Z, and H, respectively, as shown in Fig. 1c.

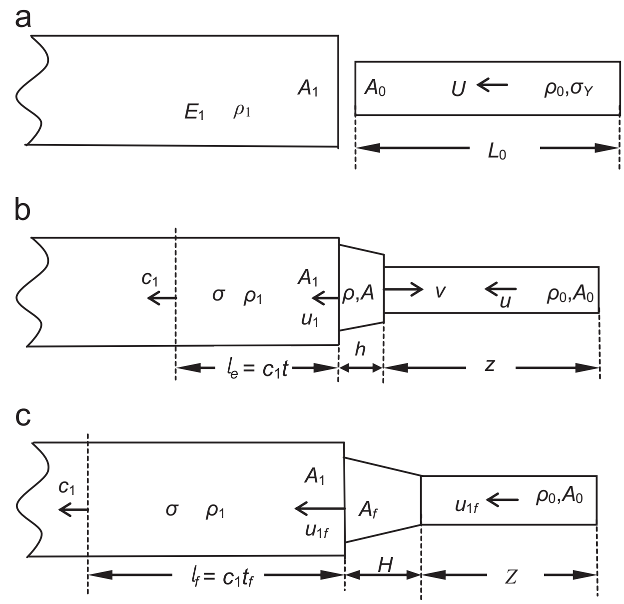

Fig. 1. A cylindrical ductile porous projectile strikes on a semi-infinite elastic bar: (a) before impact, (b) intermediate stage of deformation, and (c) final stage of deformation.

2.1. Basic equations

Similar to the Taylor model [1], one can get:

$$\frac{dh}{dt} = v \tag{1}$$

$$\frac{dz}{dt} = -(u+v-u_1) \tag{2}$$

$$\frac{du}{dt} = -\frac{\sigma_Y}{\rho_0 z} \tag{3}$$

the conservation of mass of the projectile across the plastic wave front can be written as,

$$\rho_0 A_0(u+v-u_1) = \rho A v \tag{4}$$

in the classical Taylor model, it is assumed that the cross-sectional area A_0 of the undeformed element of the projectile suddenly increases to A when passing through the plastic wave front. Qian [17] noticed that a very short period was actually needed to complete