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Transient responses of bi-layered structure based on generalized thermoelasticity: Interfacial conditions

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ABSTRACT

Multi-layered structures are always designed to apply in the harsh environment, *e.g.* high temperatures and impact loads, *etc.* In this work, thermoelastic analysis of a bi-layered structure is implemented in the context of generalized thermoelasticity, *e.g.* GL model. To distinguish from existed works, the thermal contact resistance and the elastic wave impendence at the interface are considered. Accordingly, they may degenerate into the continuous conditions by selecting ideal parameters. In the numerical part, a semi-analytical solution is obtained by using the Laplace transform method, upon which the effects of the thermal contact resistance, elastic wave impendence ratio, thermal conductivity ratio and heat capacity ratio on the responses are studied. And finally, some concluding remarks are summarized, based upon which the structure may be optimized by adopting suitable material ratio in both thermal and elastic sense.

1. Introduction

To strengthen and lighten the structure, new materials are being developed, such as ceramics or composite materials. And they are widely used in engineering practices, such as automotive, naval, and aerospace [1–3]. The stress analysis of these structures, even in thermal environment, plays a significant role in their optimization designs. If such structures are subjected to thermal shock, for example, the situation may be critical: the interface may suffer large stress, and even failure. So the behavior of thermoelasticity at the interface of layered medium is an important issue. The analysis of a multilayered domain under thermal shock is both theoretically and practically significant [4]. The classical coupled theory of thermoelasticity was developed by Biot [5], and in the thermoelastic context, some problems were considered, such as: thermoelastic contact [6,7], thermoelastic stability [8,9], and thermoelastic analysis [2,10]. However, the classical thermoelasticity predicts an infinite speed of heat propagation, which contradicts physical facts [11]. To eliminate this paradox, generalized thermoelasticity (GTE) theories were developed, which admits so-called second-sound effect, that is, finite velocity of thermal signals is predicted. Among GTE models, the widely accepted models contains:

- Lord and Shulman model (or simply denoted by LS model) [12], which was introduced by postulating a new heat conduction law to replace the classical Fourier's law. And thus, a wave-type heat conduction was obtained. In this model, time derivative of heat flux was considered, and an additional material parameter named as relaxation time was introduced.
- Green and Lindsay model (or simply denoted by GL model) [13], which was proposed by introducing the temperature rate into the constitutive equations of classical thermoelasticity, and accordingly, two relaxation times were adopted. As a result, governing equations of both mechanical field and thermal field were modified.

There are also some models, which are expected to be widely applied, such as: the GN model [14,15], inertia entropy model [16], and thermomass model [17]. Within the theoretical system, Youssef and El-Bary investigated the thermoelastic responses of layered composite material with variable thermal conductivity [18]. Wave propagation and localization in layered structures were studied under the LS model [19]. Temperature and stress distribution at the interface of an elastic layer and a rigid foundation were also presented [1]. And in the context of GN model, Yu et al. [20] analyzed the wave propagation of layered plates. The dispersion relations of thermoelastic waves were obtained by invoking continuity at the interface [21]. El-Bary and Youssef [22] considered the heat shock problems of layered composite structure under LS model, while the continuous interfacial conditions were applied. In addition, for heat conduction, an exact solution to transient heat conduction in cylindrical multilayered composites was presented by Delouei et al. [23]. Yang and Shi [24] established a stability test for heat conduction in a 1D multilayered solid.

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Akbarzadeh and Chen [25] provided a semi-analytic solution *via* Bessel functions and the Laplace inversion for 1D functionally graded media with perfect/imperfect bonding interfaces for the dual-phase-lag (DPL) and hyperbolic heat conduction theories. Sherief and Anwar [26] studied a one-dimensional thermoelastic problem of an infinitely long annular cylinder consisting of two different materials with axial symmetry in the context of generalized thermoelasticity. Abd El-Latief and Khader [27] considered a 1D problem for a half-space overlaid by a thick layer of a different material in the context of the fractional order theory of thermoelasticity.

Laminated composites often contain imperfections, such as small voids and defects at the interfaces where cracks may initiate and propagate. For this reason, the multi-physics of imperfectly bonded composites has become a subject of study [28–30]. Hatami-Marbini and Shodja [31], for example, studied the stress field of multi-phase inhomogeneity systems with perfect/imperfect interfaces under uniform thermal and far-field mechanical loading. Duan and Krihaloo [32] studied the effects of imperfect bonding between the inclusions and matrix on the effective thermal conductivity of heterogeneous media.

On the other hand, thermal contact resistance exists at the interface between two materials, which has a significant effect on the design and performance of devices with multilayer thin films, such as: superconductors and microelectronic layer packages, where heat dissipation is a crucial issue that limits the performance, reliability and further miniaturization of these devices [33]. The existence of a thermal resistance at the interface between adjacent layers results in a temperature difference at the interface. High temperature gradients can cause a thermal damage if the difference in temperature at the interface is not kept to a minimum. Hence, the choice of the materials with the proper thermal properties those result in a minimum interfacial temperature difference (and hence minimum interfacial temperature gradient) becomes crucial in the design of thin film structures particularly when the designer does not have much control over the reduction of the thermal boundary resistance [34]. Li and Cheng [35–37] studied the thermal shock resistance of ceramics. Lor and Chu [38] used the thermal wave model to study the effect of the interfacial thermal resistance on heat transfer in a composite medium using a radiation boundary condition at the interface. The thermal wave model was used by Khadrawi et al. [39] to study the thermal behavior of perfect and imperfect contact composite slabs with a constant interfacial thermal resistance. Akbarzadeh and Pasini [40] studied the thermal responses of one-dimensional multilayered systems, functionally graded solid media, and porous materials under alternative heat conduction theories. To display the variations of the stress and temperature fields, a step-by-step algorithm was proposed by Atarashi and Minagawa [41] to give the solutions in each layer of the plate, under the boundary conditions at the outer surfaces and the interfaces between layers, but the mechanical bonding was perfect.

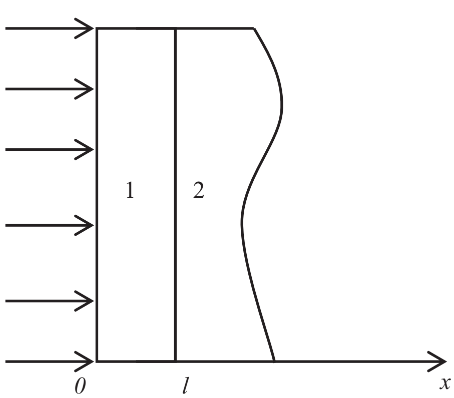

Fig. 1. Schematic diagram of the bi-layered structure.

It appears that the transient response of 1D layered structures with thermal contact resistance and elastic wave impendence has not been studied. The effects of the interfacial conditions on the temperature and displacement distribution seem unclear. To address this deficiency, a 1D bi-layered model with thermal contact resistance and elastic wave impendence is considered in the context of GL model in this work. And we provide a semi-analytical solution *via* the Laplace transform method. By numerical implementation, the effects of the interfacial conditions on the thermoelastic responses are evaluated. In Section 2, the problem considered and the governing equations are introduced. Section 3 is devoted to the numerical method adopted in this work. Numerically calculated results are depicted, where detailed discussions are provided in Section 4. And finally, some conclusions are given.

2. Problem formulation and governing equations

The model considered in this work is shown in Fig. 1. The model is made up of two layers with different materials, which are in contact with each other at the location $x=l$. Medium 2 is assumed to be infinite to neglect the reflection from the right side of medium 2. The left surface of medium 1 is subjected to a sudden heating, while in the mechanical sense it is traction free. The infinite (or the right side of medium 2) is undisturbed, that is, no displacement occurs and the temperature keeps initial value. It is noted that in this work the thermal contact resistance, reflection and transmission behavior of elastic wave are considered at the interface. To simplify the analysis, the problem can be considered as one-dimensional case. And in this case, the equation of motion for the jth medium has the form (Throughout this paper, the sub-symbols $j=1$ and 2 indicate the variables in the jth medium):

$$\frac{\partial \sigma_j}{\partial x}=\rho_j\frac{\partial^2 u_j}{\partial t^2} \tag{1}$$

And the equations of local energy balance and heat conduction are as:

$$\frac{\partial q_j}{\partial x}+\rho_j T_0\frac{\partial \eta_j}{\partial t}=0 \tag{2}$$

$$k_j\frac{\partial \theta_j}{\partial x}+q_j=0 \tag{3}$$

with the following constitutive equations:

$$\sigma_j=(\lambda_j+2\mu_j)\frac{\partial u_j}{\partial x}-\gamma_j(\theta_j+\tau_{j1}\frac{\partial \theta_j}{\partial t}) \tag{4}$$

$$\rho_j\eta_j=\gamma_j\frac{\partial u_j}{\partial x}+\frac{\rho_j c_{Ej}}{T_0}(\theta_j+\tau_{j2}\frac{\partial \theta_j}{\partial t}) \tag{5}$$

where σ_j is the stress component in the x-direction, q_j is the heat-flux, η_j is the entropy density, ρ_j is the mass-density, k_j is the thermal conductivity, c_{Ej} is the heat capacity, λ_j and μ_j are Lame's constants, $\gamma_j=(3\lambda_j+2\mu_j)\alpha_{tj}$ is the thermo-elastic coupling coefficient, and α_{tj} is the linear thermal expansion coefficient. Eq. (1) may be further updated with consideration of (4):

$$\rho_j\frac{\partial^2 u_j}{\partial t^2}=(\lambda_j+2\mu_j)\frac{\partial^2 u_j}{\partial x^2}-\gamma_j(\frac{\partial \theta_j}{\partial x}+\tau_{j1}\frac{\partial^2 \theta_j}{\partial x\partial t}) \tag{6}$$

And similarly, combining Eqs. (2), (3) and (5), one may obtain the governing equation for temperature as:

$$k_j\frac{\partial^2 \theta_j}{\partial x^2}=\gamma_j T_0\frac{\partial^2 u_j}{\partial x\partial t}+\rho_j c_{Ej}(\frac{\partial \theta_j}{\partial t}+\tau_{j2}\frac{\partial^2 \theta_j}{\partial t^2}) \tag{7}$$