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# Role of rough surface on contact between magneto-electro-elastic materials and orthotropic solid

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A B S T R A C T

The solid materials' surfaces will be found to be rough if magnified sufficiently, no matter how flat they may superficially appear to be. This article establishes a model to examine the effect of rough surface on the contact behavior of magneto-electro-elastic materials with partial or full contact with an orthotropic solid pressed by external loading. The lack between the two surfaces is featured by the gap at the peaks of the rough surfaces. It is found that when the external loading is far less than the critical loading, which is the minimum loading to approach the full contact, the relationship between the contact width and external loading, and the surface contact distribution have the same form as between two purely elastic cylinders in the theory of Hertzian contact. Numerical experiments are done to demonstrate how the component volume fraction of magneto-electro-elastic materials affects the contact behavior. Numerical results show that the contact region between magneto-electro-elastic materials and orthotropic solid is dominated by the external loading other than the component volume fraction of magneto-electro-elastic materials.



## 1. Introduction

When two solids possessing highly polished surfaces are pressed into contact, the real contact area will be irregular and usually only is a small fraction of the nominal contact area, named rough surface. Surface roughness plays an important part in understanding the nature of machined surfaces and needs to be understood to improve the functional performance of many engineering components (Bhushan, 1998; Lane, 2002; Carbone et al., 2004; Persson, 2006; Buzio and Valbusa, 2008). Revealing interaction of real rough surfaces in contact becomes an interesting and difficult issue.

Many contributions to the area of regularly wavy contact have been made by employing various methods and in many different contexts for purely elastic materials and structures. Westergaard (1939) first derived an explicit solution for the contact problem of an elastic half-space with a wavy surface, and presented expressions for the surface tractions and relationship between the applied pressure and the contact area. Johnson et al. (1985) expanded the elastic contact of a one-dimensional sinusoidal surface presented by Westergaard (1939) to that of a two-dimensional sinusoidal surface with a flat, and found the variation of the mean separation with load. Since rough surfaces usually make partial contact when compressed (Bowden and Tabor, 1954), reduction of true contact area due to partial contact for rough surfaces and its relation with adhesion and friction have drawn researcher's attention. Kaczyfiski and Matysiak (1988) obtained a certain class of solutions presenting the stress distributions in the microperiodic two-layered half-space due to rigid punches in the presence of friction and perfect contact, and revealed the micro-local effects, i.e. the effects due to the periodic structure of the body by means of microlocal parameters. Nosonovsky and Adams (2000) studied dry, steady-state frictional sliding of two elastic bodies, one of which has a periodic wavy surface, and analyzed the dependence of the true contact area on loading. Cai and Lu (2000) concerned sliding contact with Coulomb friction under a rigid punch with a flat, inclined or circular profile. Block and Keer (2008) proposed an approach based upon the summation of evenly spaced Flamant solutions to study various contact problems of elastic solid with regularly periodic surfaces including sliding contact, complete stick and partial slip. Recently, Soldatenkov (2011) examined the contact of a wavy punch with an elastic strip in the presence of friction and wear. These contact problems with rough surface were all for elastic materials.

The progress in the field of materials science has greatly affected the progress of human civilization. As smaller components are urgently needed with the rapid improvement of mobile devices, a kind of smart materials possessing a peculiar magneto-electric (ME) coupling effect at room temperature, named as “magneto-electro-elastic materials” or “piezomagnetic/piezoelectric materials”, is

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designed (Li et al., 2011). A great number of research activities have been carried out to illustrate the coupling between electric and magnetic orders and expand potential applications in novel multi-functional devices (Islam and Priya, 2006; Lokare et al., 2008; Vaz et al., 2010). For more details, please refer to Ma et al. (2011) and Kambale et al. (2012). From the point of view of mechanics, non-periodic contact problems involving a semi-infinite magneto-electro-elastic medium under a rigid punch with various profiles were concerned (Zhou and Lee, 2012a, 2012b, 2013; Elloumi et al., 2013; Li et al., 2014; Ma et al., 2014, 2015). In practical engineering, magneto-electro-elastic materials can be bonded to the surface of or embedded within a host structure, and there exists rough surface. Thus, contact problem of magneto-electro-elastic materials with rough surface needs to be addressed.

The present work proposes a theory to address contact problem of magneto-electro-elastic materials in contact with orthotropic solid with rough surfaces by using the potential theory and dual series equation methods. The gap at the peaks of the rough surfaces is used to feature the lack between the two surfaces. Dual series equations are obtained and then reduced to an integral equation of the Abel type that is solved exactly. In case of small external loading, quantity that describes the mismatch in curvatures in two sinusoidal profiles at the point of initial contact is defined; and it is seen that the relationship between the contact width and external loading, and the surface contact distribution have the same form as those for two purely elastic cylinders in the theory of Hertzian contact. Numerical results depict that compared with the component volume fraction of magneto-electro-elastic materials, the external loading plays a key role in determining the contact region between magneto-electro-elastic materials and orthotropic solid.

## 2. Problem statement and boundary conditions

Consider that magneto-electro-elastic materials and an orthotropic solid contact with slightly wavy surface possessing periodic profiles by uniform pressure $p$ at infinity. Magneto-electro-elastic materials and the orthotropic solid contact are in partial contact. The two wavy surfaces of magneto-electro-elastic materials and an orthotropic solid hold the following relationship:

$$w^{(1)}(x,0)-w^{(2)}(x,0)=\frac{1}{2}\Gamma_0[-1+\cos(x)], \qquad 0\le|x|<a, \tag{1}$$

where $w(x,z)$ is the displacement along $z$-axis, the superscript 1 denotes the orthotropic solid and 2 magneto-electro-elastic materials. The quantity $\Gamma_0$ is the gap at the peaks of the wavy surfaces (Fig. 1). In Eq. (1), $a$ is the unknown half length of the contact region. There are two reasons why a cosine function is used to describe the roughness of the contact surface in Eq. (1). On the one hand, the cosine wave, an even function used to describe the difference of the vertical displacements, represents the first term in a Fourier decomposition of an actual rough surface. On the other hand, after differentiating to exclude the rigid body displacement, a sine function can be obtained in one equation of the dual series equation as will be seen later in Eq. (61), which benefits one to get the exact solution. Note that if needed, the wave length $2\pi$ can be scaled as $2M$.

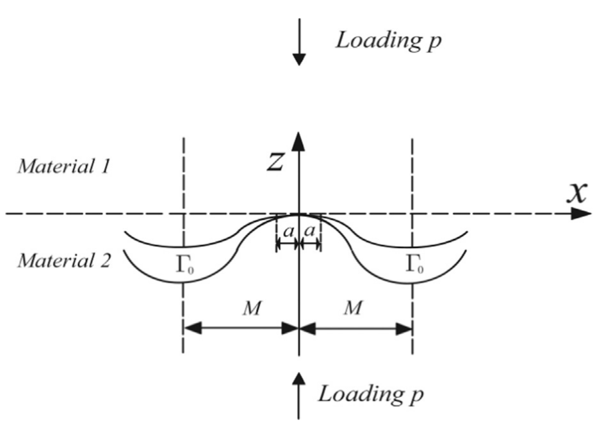


**Fig. 1.** Partial contact between two rough surfaces.

Considering that the piezoelectric materials and the orthotropic solid are pressed to contact by uniform pressure $p$ at infinity, one can determine unknown half length $a$ with the consideration of the following conditions:

$$\sigma_{zz}^{(1)}(x,+\infty)=-p, \tag{2}$$

$$\sigma_{zz}^{(2)}(x,-\infty)=-p, \tag{3}$$

where $\sigma_{zz}$ denotes the normal stress.

As seen in Eq. (1), periodic profile with wave length $2\pi$ is modeled. Thus, for the present periodic contact problem, periodicity conditions on the lines $x=\pm n\pi$ should also be satisfied

$$u^{(1)}(\pm n\pi,z)=u^{(2)}(\pm n\pi,z)=0, \tag{4}$$

$$\sigma_{xz}^{(1)}(\pm n\pi,z)=\sigma_{xz}^{(2)}(\pm n\pi,z)=0, \tag{5}$$

$$D_x(\pm n\pi,z)=0, \tag{6}$$

$$B_x(\pm n\pi,z)=0, \tag{7}$$

where $u(x,z)$ is the displacement along $x$-axis, $\sigma_{xz}$ denotes the shear stress, and $D_\beta$ and $B_\beta(\beta=x,z)$ represent the electric displacement components and magnetic induction components of magneto-electro-elastic materials.

On the interface $z=0$, stresses are continuous

$$\sigma_{zz}^{(1)}(x,0)=\sigma_{zz}^{(2)}(x,0)\le 0, \qquad -\pi\le x\le\pi, \tag{8}$$

$$\sigma_{xz}^{(1)}(x,0)=\sigma_{xz}^{(2)}(x,0)=0, \qquad -\pi\le x\le\pi, \tag{9}$$

Outside the contact region, the normal stresses are free

$$\sigma_{zz}^{(1)}(x,0)=\sigma_{zz}^{(2)}(x,0)=0, \qquad a<|x|\le\pi. \tag{10}$$

The interface between magneto-electro-elastic materials and the orthotropic solid is electrically insulating and magnetically insulating, i.e.

$$D_z(x,0)=0, \qquad -\pi\le x\le\pi, \tag{11}$$

$$B_z(x,0)=0, \qquad -\pi\le x\le\pi. \tag{12}$$

## 3. Basic equations

The equilibrium equations for the linear magneto-electro-elastic materials and the orthotropic solid free of any body sources take the form

$$\left.\begin{array}{l}\frac{\partial\sigma_{xx}^{(j)}}{\partial x}+\frac{\partial\sigma_{xz}^{(j)}}{\partial z}=0\\ \frac{\partial\sigma_{xz}^{(j)}}{\partial x}+\frac{\partial\sigma_{zz}^{(j)}}{\partial z}=0\end{array}\right\} \qquad (j=1,2). \tag{13}$$

For magneto-electro-elastic materials, the Gauss equation of electricity and the Maxwell equation of magnetism are given as

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_z}{\partial z}=0, \tag{14}$$

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_z}{\partial z}=0. \tag{15}$$