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Elastodynamic analysis of carbon nanotube-reinforced functionally graded plates

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ABSTRACT

In this paper, elastodynamic problem of carbon nanotube-reinforced functionally graded (CNTR-FG) plates is studied using the element-free kp-Ritz method based on first order shear deformation theory (FSDT). The plates are reinforced by single-walled carbon nanotubes (SWCNTs) with different types of distributions, i.e. uniform and three kinds of functionally graded distributions of carbon nanotubes along thickness direction of the panels. Extended rule of mixture is employed to estimate effective material properties of the resulting nanocomposites. Two-dimensional displacement fields of the plates are approximated by a set of mesh-free kernel particle functions. The discretized governing equations are developed via the Ritz procedure, and numerical time integration is performed through the Newmark-β method. Convergence studies are carried out by considering the influence of support size and number of nodes to verify the numerical stability of the present element-free method. Numerical simulations are used to study the effect of carbon nanotube volume fraction, plate width-to-thickness ratio, plate aspect ratio, boundary condition, load type and distribution type of carbon nanotubes on the dynamic responses of CNTR-FG plates.

1. Introduction

Carbon nanotubes (CNTs) have been demonstrated to have excellent mechanical, electrical and thermal properties. Due to these remarkable properties, such as high strength, high stiffness and high aspect ratio but low density, CNTs can be considered as an excellent reinforcement for polymer composites that may significantly improve the mechanical, electrical and thermal properties of the resulting nanocomposites. Taking advantage of their conductivity and high aspect ratios, Kilbride et al. [1] produced conductive plastics with exceedingly low percolation thresholds and measured alternating current and direct current conductivities in polymer-nanotube composite thin films. Biercuk el al. [2] used single-walled carbon nanotube (SWCNT) to augment thermal transport properties of industrial epoxy. The results suggested that thermal and mechanical properties of SWCNT-epoxy composites were improved without the need to chemically functionalize the carbon nanotubes.

Earlier studies on CNT-reinforced composite materials were mainly focused on material properties and constitutive modeling. Shaffer and Windle [3] first investigated the thermo-mechanical and electrical properties of carbon nanotube reinforced composite films based on the formation of a stable colloidal intermediate. Dalmas et al. [4] analyzed linear and nonlinear mechanical behaviors and electrical properties of multi-walled carbon nanotube/polymer nanocomposites. Liu et al. [5] studied morphology and mechanical properties of multi-walled carbon nanotubes reinforced Nylon-6 composites. The results showed the elastic modulus and the yield strength of the nanotube reinforced composites are improved by about 214% and 162%, respectively, by incorporating only 2 wt % MWNTs. Meincke et al. [6] examined mechanical properties and electrical conductivity of composites of polyamide-6 and carbon nanotubes. Tensile tests showed a significant increase of 27% in the Young's modulus, however, elongation at break of these materials dramatically decreases due to embrittlement of the polyamide-6. Xu et al. [7] studied mechanical properties and interfacial characteristics of carbon nanotube reinforced epoxy thin films. With only 0.1 wt% MWNTs added, they found a 20% increase in elastic modulus. Cadek et al. [8] studied the morphological and mechanical properties of

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arc-discharge MWCNT in both polyvinyl alcohol and polyvinyl carbazole. Their results showed an increase in modulus from 7 GPa to 12.6 GPa on addition of 0.6 vol% MWCNT in polyvinyl alcohol. Pötschke et al. [9] examined the rheological behavior of composite of polycarbonate and CNTs using oscillatory rheometry at 260 °C. The results revealed that 2 wt% nanotubes caused significant improvement in electrical resistivity and complex viscosity.

In traditional nanocomposites, the resulting mechanical, thermal or physical properties do not vary spatially at the macroscopic level because nanotubes are distributed either uniformly or randomly in the composites. Functionally graded materials (FGMs) are a new breed of composite materials with properties that vary spatially according to a certain non-uniform distribution of the reinforcement. Stimulated by the concept of FGMs, the pattern of the functionally graded (FG) distribution of reinforcement has been successfully applied for CNT-reinforced composite materials. CNT-based FGMs were first proposed by Shen [10] with CNT distributions within an isotropic matrix designed purposefully to grade with certain rules along the desired directions to improve the mechanical properties of the structures. Then, a series of investigations were carried out to study the dynamic properties of CNT-RC materials. Heshmati and Yas [11] studied the dynamic behaviors of FG-MWCNT-polystyrene nanocomposite beams subjected to multi-moving loads. Rafiee et al. [12] studied the nonlinear dynamic stability of initially piezoelectric FG-CNTRC plates. Shooshtari et al. [13] examined the thermo-mechanical nonlinear dynamic characteristics of FG-CNTRC plates using the multi-scale approach. Dynamic analysis of functionally graded nanocomposite beams under the actions of moving load was presented by Yas and Heshmati [14]. By using the element-free kp-Ritz method, Lei et al. [15] carried out dynamic stability analysis of carbon nanotube-reinforced functionally graded (CNTR-FG) cylindrical panels under static and periodic axial force. Rasool el al. [16] carried out stress wave propagation analysis of nanocomposite cylinders subjected to an impact load. Wang et al. [17] presented an analysis of impact response of FG-CNTRC structures under thermal conditions. Hassan et al. [18] presented the transient dynamic analysis of displacement field and elastic wave propagation for functionally graded nanocomposites using local integral equations (LIEs).

This paper investigates the dynamic behavior of carbon nanotube-reinforced functionally graded (CNTR-FG) plates using the element-free kp-Ritz method based on FSDT. Single-walled carbon nanotubes (SWCNTs) are selected as reinforcements and different types of distributions, i.e. uniform and three kinds of functionally graded distributions are considered. The Newmark-β method is employed for integration of the time history for the elastodynamic problem. The essential boundary conditions are imposed by the full transformation method. The stability and accuracy of the present element-free kp-Ritz method is demonstrated through convergence studies. In computational simulation, several numerical examples are provided to investigate the influence of carbon nanotube volume fraction, plate width-to-thickness ratio, plate aspect ratio, boundary condition, load type and distribution type of CNTs on the dynamic responses of CNTR-FG plates.

2. Energy formulation

The displacement field of the first-order shear deformation theory (FSDT) is of the form [19]

$$u(x,y,z) = u_0(x,y) + z\phi_x(x,y), \tag{1}$$

$$v(x,y,z) = v_0(x,y) + z\phi_y(x,y), \tag{2}$$

$$w(x,y,z) = w_0(x,y), \tag{3}$$

where u, v and w are the displacement components in x, y and z directions, respectively, and (u_0, v_0, w_0, ϕ_x, ϕ_y) are the displacement components on the mid-plane ($z=0$), and

$$\phi_x = \frac{\partial u}{\partial z},\ \phi_y = \frac{\partial v}{\partial z}, \tag{4}$$

The strain components are given by:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa},\ \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \boldsymbol{\gamma}_0 \tag{5}$$

where

$$\boldsymbol{\varepsilon}_0 = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix},\ \boldsymbol{\kappa} = \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \end{Bmatrix},\ \boldsymbol{\gamma}_0 = \begin{Bmatrix} \phi_y + \frac{\partial w_0}{\partial y} \\ \phi_x + \frac{\partial w_0}{\partial x} \end{Bmatrix}. \tag{6}$$

Then, the constitutive relations are expressed as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} - \begin{Bmatrix} \alpha_{11} \\ \alpha_{22} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \Delta T \right), \tag{7}$$

where

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}},\ Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}},\ Q_{12} = \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}}, \tag{8}$$

$$Q_{66} = G_{12},\ Q_{44} = G_{23},\ Q_{55} = G_{13}, \tag{9}$$

and ΔT is the temperature change with respect to a reference state. E_{11} and E_{22} are effective Young's moduli, G_{12}, G_{13} and G_{23} are the shear moduli, ν_{12} and ν_{21} are Poisson's ratios and α_{11} and α_{22} are thermal expansion coefficients.

The relation between the stress resultants and the strains is given as:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{Q}s \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \overline{\mathbf{B}} & 0 \\ \overline{\mathbf{B}} & \mathbf{D} & 0 \\ 0 & 0 & \mathbf{A}^s \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}_0 \\ \boldsymbol{\kappa} \\ \gamma_0 \end{Bmatrix} - \begin{Bmatrix} \mathbf{N}^T \\ \mathbf{M}^T \\ 0 \end{Bmatrix}, \tag{10}$$

where the in-plane force resultants, moment resultants, transverse force resultants and thermal stress resultants are given as:

$$\mathbf{N} = \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \mathrm{d}z,\ \mathbf{M} = \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z\mathrm{d}z,$$

$$\mathbf{Q}s = \begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} dz, \tag{11}$$

$$\mathbf{N}^T = \int_{-h/2}^{h/2} \begin{bmatrix} \alpha_{11} & \alpha_{22} & 0 \end{bmatrix} (Q_{11}+Q_{12})\Delta T \mathrm{d}z,$$

$$\mathbf{M}^T = \int_{-h/2}^{h/2} \begin{bmatrix} \alpha_{11} & \alpha_{22} & 0 \end{bmatrix} (Q_{11}+Q_{12})\Delta T z dz. \tag{12}$$

the extensional $\mathbf{A}$, coupling $\overline{\mathbf{B}}$, bending $\mathbf{D}$ and transverse shear $\mathbf{A}^s$ stiffness are

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2)\mathrm{d}z,\ A_{ij}^s = K\int_{-h/2}^{h/2} Q_{ij} dz \tag{13}$$

where K denotes the transverse shear correction coefficient, and is suggested to be $K = 5/(6-(v_1V_1+v_2V_2))$ for functionally graded materials [20].