International Journal of Mechanical Sciences 99 (2015) 218–227

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Thermoelastic instability of a functionally graded layer interacting with a homogeneous layer

Jia-Jia Mao, Liao-Liang Ke*, Yue-Sheng Wang

Institute of Engineering Mechanics, Beijing Jiaotong University, Beijing 100044, PR China

A R T I C L E I N F O

Article history:
Received 23 January 2015
Received in revised form
16 May 2015
Accepted 21 May 2015
Available online 29 May 2015

Keywords:
Thermoelastic instability
Functionally graded materials
Contact mechanics
Critical heat flux

A B S T R A C T

The interaction between the thermoelastic distortion and pressure-dependent thermal contact resistance can cause the themoelastic instability when the conductive heat transfers between two elastic bodies in the static frictionless contact. Using the perturbation method, this paper investigates the thermoelastic instability of a system consisting of a functionally graded material (FGM) layer and a homogeneous layer under the plane strain state. The two layers are pressed together by a uniform pressure, and transmit a uniform heat flux at their interface. The material properties of the FGM layer are assumed to be of exponential variation along the thickness direction. The thermoelastic stability behavior of five types of material combinations depending on the ratio of their material properties are discussed in details. The results imply that the FGMs can be used to improve the thermoelastic contact stability of the systems.

1. Introduction

The thermoelastic deformations caused by the conductive heat transfer between two dissimilar contacting bodies could affect the contact pressure distribution and the extent of the contact area, which will generally affect the boundary conditions of the heat conduction problem in return. As a result, the thermoelastic instability may occur between the contacting bodies. Generally, the thermoelastic instability problem can be identified as two distinct sorts, i.e., the frictionally-excited thermoelastic instability [1] and the static thermoelastic instability [2,3], either because of the frictional heating or because of the pressure-dependent thermal contact resistance.

The frictionally-excited thermoelastic instability implies that the sliding contact system involves a frictional heating. Instabilities due to the frictional heating are found in energy dissipating systems, such as brakes and crutches. Dow and Burton [4] investigated the effect of material properties on the frictionally-excited thermoelastic instability for the case of a scraper or blade sliding normal to its line of contact on a thermally conductive semi-infinite body. Burton et al. [5] analyzed the problem of two sliding flat plates contacting on a straight common edge under a pressure perturbation on the interface. Using the perturbation method, Lee and Barber [6] evaluated the influence of finite disk thickness on the stability behavior of an automotive disk brake through a model of a finite thickness layer sliding between two half-plane. Yi et al. [7] developed the finite element method to reduce the thermoelastic instability problem for a brake disk to an eigenvalue problem with the critical speed. They explored the effect of the geometric complexity on the critical speed and the associated mode shape. Yi et al. [8] further determined the critical sliding speed for the thermoelastic instability of an axisymmetric clutch or brake. Lee [9] used a finite layer model with one-sided frictional heating to study the instability in automotive drum brake systems. He also performed some vehicle tests to observe the critical speeds of the drum brake systems with aluminum drum materials. Decuzzi et al. [10] studied thermoelastic instability of a two-dimensional model of a clutch/brake with an infinite number of disks. They analyzed the influence of the disk thickness ratio on the critical speed, critical wave parameter and migration speed of the system.

In the static thermoelastic contact, if the heat conducts across the interface between two bodies, the thermoelastic instability could be caused by the interaction of the thermoelastic distortion and pressure-dependent thermal contact resistance. The static thermoelastic instability is visual for many industrial settings, i.e., castings, molding, valves, pistons, thermostats, thermal expansion of railways, cylinder heads, etc. [11]. Barber [12] analyzed the unstable of two half-planes under a sinusoidal contact pressure perturbation on a nominally uniform pressure for sufficiently large

* Corresponding author. Tel.: +86 10 51685755; fax: +86 10 51682094.
E-mail address: llke@bjtu.edu.cn (L.-L. Ke).

http://dx.doi.org/10.1016/j.ijmecsci.2015.05.018
0020-7403/

heat fluxes. Zhang and Barber [13] investigated the influence of material properties on the stability criterion for the thermoelastic contact between two half-planes. They classified the material combinations into five categories depending upon the ratios of the thermal conductivities, diffusivities and distortivities. Using Zhang and Barber's classification [13], Yeo and Barber [14] considered the effect of a finite geometry on the stability in the system of a layer and a half-plane. Li and Barber [15] discussed the thermoelastic stability of a system consisting of two layers in contact using the perturbation method. Furthermore, the static thermoelastic instability problem was also investigated by Schade et al. [16] for two bonded half-plane and Schade and Karr [17] for a layer bonded to a half-plane.

Specially, Ciavarella and his co-authors presented comprehensive studies on the thermoelastic instability problems by considering the combined effects of the pressure-dependent thermal contact resistance and frictional heating. Ciavarella et al. [18] considered thesliding of a one-dimensional rod against a rigid plane with frictional heating and a pressure-dependent thermal contact resistance. Afferrante and Ciavarella [19,20] studied an elastic conducting half-plane sliding against a rigid perfect conductor wall or two half-planes sliding out-of-plane by considering the combined effects. Afferrante and Ciavarella [21] analyzed the Aldo model by introducing the effect of frictional heating. Ciavarella and Barber [22] concerned the stability boundary for the thermoelastic contact of a rectangular elastic block sliding against a rigid wall in the presence of the thermal contact resistance.

Functionally graded materials (FGMs) are usually a mixture of two distinct material phases with continuously varying volume fractions of constituent materials, hence their effective material properties change in a continuous and smooth manner. FGMs used as coatings or interfacial zones can reduce the magnitude of residual and thermal stresses, mitigate stress concentration, increase fracture toughness, and resist the contact damage [23–31]. Recently, the thermoelastic instability problems of FGMs were also concerned by many investigators due to their potential application to improve thermolastic stability behaviors in the brake disk system. For the frictionally-excited thermoelastic instability, Jang and his co-authors presented comprehensive works, such as, a stationary FGM layer between two sliding homogeneous layers [32], an FGM half-plane sliding against a homogeneous half-plane [33] and an FGM layer sliding against two homogeneous half-plane [34]. Their studies showed that FGMs could improve the contact stability in the frictional sliding system through an optimal gradient index of FGMs. Hernik [35] dealt with the study of the global thermoelastic instability of a brake disk made of either the isotropic homogeneous metal matrix composite or FGMs. For the static thermoelastic instability, Mao et al. [36] investigated the stability behaviors for three types of material combinations between an FGM layer and a homogeneous half-plane. The results showed that the finite geometry and the gradient index of the FGMs have a great influence on the static thermoelastic instability behavior of them systems.

In this paper, we discuss the static thermoelastic instability between an FGM layer and a homogeneous layer under the plane strain state by using the perturbation method. The two layers are pressed together by a uniform pressure, and transmit a uniform heat flux at their interface. The thermoelastic properties of the FGM layer, such as the shear modulus, thermal conductivity coefficient, thermal expansion coefficient, specific heat, density, are assumed to be exponential along the thickness direction. Because of the imperfect contact between these two layers, a pressure-dependent thermal contact resistance is considered at the interface, which depends on the ratios of the material properties of two layers. The effects of the gradient index and layer thickness on the critical heat flux are discussed for five types of material combinations.

2. Formulation of the thermoelastic instability problem

Fig. 1 shows the frictionless contact between an FGM layer ($0 \le y \le h_1$) and a homogeneous layer ($-h_2 \le y < 0$) at their common surface $y = 0$ where h_1 and h_2 are the thickness of the FGM layer and homogeneous layer, respectively. The two layers are pressed together by a uniform pressure p_0. A uniform heat flux $q_y = q_0$ is imposed at the top surface $y = h_1$ and bottom surface $y = h_2$, flowing across their common interface in the positive y-direction. The thermoelastic properties of the FGM layer are assumed to be the exponential forms as

$$\mu(y) = \mu_b e^{\beta y}, \beta = \ln(\mu_t/\mu_b)/h_1, \tag{1a}$$

$$k(y) = k_b e^{\delta y}, \delta = \ln(k_t/k_b)/h_1, \tag{1b}$$

$$\alpha(y) = \alpha_b e^{\gamma y}, \gamma = \ln(\alpha_t/\alpha_b)/h_1, \tag{1c}$$

$$c(y) = c_b e^{\varepsilon y}, \varepsilon = \ln(c_t/c_b)/h_1, \tag{1d}$$

$$\rho(y) = \rho_b e^{\varsigma y}, \varsigma = \ln(\rho_t/\rho_b)/h_1, \tag{1e}$$

where $\mu(y)$, $k(y)$, $\alpha(y)$, $c(y)$ and $\rho(y)$ are the shear modulus, thermal conductivity coefficient, thermal expansion coefficient, specific heat, density, respectively; β, δ, γ, ε and ς are the gradient indexes; the Poisson's ratio v is assumed as a constant for simplicity; subscripts "b" and "t" refer to the bottom and top of the FGM layer, respectively.

It should be pointed that the metal and ceramic phases are mixed in different volume ratios to form the multi-layered structures in the real FGMs. It can be fabricated by powder metallurgy, chemical vapor deposition, plasma spraying process, centrifugal method [37]. The real material profile of FGMs is of step changes across the thickness, which is often modeled as the linear function or power-law function [38–40]. However, the contact and crack problems of FGMs for linear case or power-law case are quite difficult to obtain the analytical solutions. Therefore, most investigators tend to use the exponential model to analyze the contact and crack problems of FGMs because the exponential model can solve the problems analytically. By using this simple model, we can get an insight on the thermoelastic stability behavior of FGMs. The results may have the potential application on improving the contact stability of the systems. That is why the exponential model is used in the present paper.

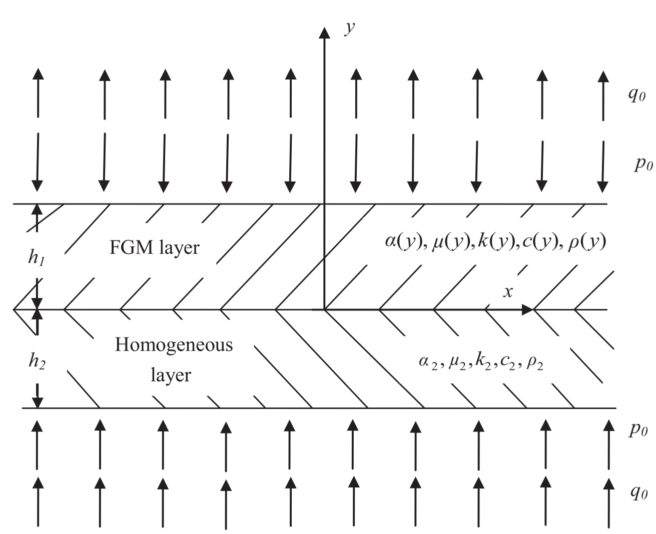

Fig. 1. An FGM layer on a homogeneous layer pressed by a uniform pressure and transmitting heat.