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Mean compressive stress constitutive equation and crashworthiness optimization design of three novel honeycombs under axial compression

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ABSTRACT

Taking typical type of honeycomb as research object, three types of novel honeycombs are studied on the aspects of theoretical prediction under axial crushing. The simplified super folding element (SSFE) theory is applied to estimate the energy dissipation of the typical elements. Taking the inertia effects into account, dynamic mean compressive stress calculating equation is further deduced by utilizing the dynamic enhancement coefficient. In order to validate these theoretical solutions, LS-DYNA is used to simulate the axial loading of three novel honeycombs. The analytical results correlate with these simulation results in an ideal manner. During the multi-objective optimization deign, optimal Latin hypercube design (OLHD) method is used to select sampling points in the design space. Meanwhile, the response surface method (RSM), as an accurate surrogate modeling method, is adopted to obtain a maximum specific energy absorption per mass (SEA_m) capacity and minimum peak crushing stress (σ_{peak}). These results yielded from the optimization indicate that the bending cellular honeycomb (BC-H) has the best energy absorption performance under the same limitation of σ_{peak}. Furthermore, the theoretical equations proposed are utilized to validate the simulation results of three optimal structures. The theoretical predictions show excellent agreement with these simulation values, which hence illustrate the feasibility and efficiency of the crashworthiness optimization method based on surrogate models and finite element analysis techniques.

1. Introduction

Nowadays thin-walled multi-cell structures, due to whose excellent energy absorption capacity, have been applied in a wide range of fields, including aerospace, marine and railway engineering. To better understand mechanical properties and evaluate the energy absorption performance of such structures, the mean crushing stress needs to be calculated first, as it determines the energy dissipation when the structure is compacted. Extensive efforts were conducted to study the dynamic response and develop theoretical models to predict the dynamic plateau strength for multi-cell structures. For examples, McFarland [1] first derived a semi-empirical model to calculate the mean crushing strength of hexagonal cell structures. Gibson et al. [2] and Gibson and Ashby [3] developed another theoretical model for honeycombs, which was then validated by experimental data. De Oliveira and Wierzbicki [4], Wierzbicki [5], Wierzbicki and Abramowicz [6] introduced a super folding theory (SFE) for predicting the plateau stress of thin-walled structures under different loading conditions. These theoretical predictions were experimentally validated by Abramowicz and Jones [7,8], Langseth and Hopperstad [9]. Wu and Jiang [10] compared the predictions based on the theory by Wierzbicki with the quasi-static compression test results on honeycombs. They found that the theoretical predictions underestimated the experimental data. Zarei Mahmoudabadi and Sadighi [11–13] further improved Wierzbicki's SFE theory by employing a more detailed geometric change during the structural deformation.

Chen and Wierzbicki [14] simplified the super folding element (SSFE) theory and improved its performance. This simplified model was then utilized by Zhang et al. [15], who successfully predicted the mean crushing strength of multi-cell square columns. Alavi Nia and Parsapour [16] studied the mechanical properties of multi-cell square tubes through experiment and numerical simulation. The theoretical prediction deduced by Zhang et al. [15] was modified. Luo et al. [17] derived a theoretical solution for the mean compressive stress of honeycomb metal under out-of-plane compression, and proved the correctness of the constitutive equation by a series of tests. Yin [18,19] extended the SSFE theory to develop a new method to calculate the plateau stress of honeycombs subjected to axial impacting. Tran and Hou

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[20,21] adopted this theory to deduce mean crushing force of multi-cell square, triangular tubes and angle element structures. Based on test data and existing theoretical models, Bai et al. [22] developed a new analytical model to predict the crushing behavior of hexagonal multi-cell thin-walled structures, e.g. honeycombs under quasi-static loading.

As an energy absorber, the multi-cell thin-walled structure is designed to absorb as much striking energy as possible per unit mass or volume. With the development of finite element (FE) methods, FE solutions or a surrogate model technology were widely used in crashworthiness design of the multi-cell structure. The design of experiments (DOE) was used to establish the approximate model by Hou et al. [23] in the crashworthiness of standard hexagonal honeycombs with different cross sections. Esfahlani et al. [24] identified the performance of various meta-models to optimize the energy absorption characteristic of hexagonal honeycomb. Li et al. [25] found the most optimized alternative square honeycomb structure via employing the RSM in crashworthiness design. The radial basis functions (RBF), Kriging, multivariate adaptive regression splines (MARS), and support vector regression (SVR) were utilized by Yin et al. [26] to predict *SEA* and $\sigma peak$ in optimization of honeycomb structure. They pointed out that different meta-models may provide rather different modeling accuracies and response values. However, there is seldomly a combination study of theory, data analysis and optimization design for honeycomb.

Meanwhile, most of these previously mentioned mean crushing strength predictions or crashworthiness optimizations were focused on the commonly used honeycombs. As higher designing requirements being placed, more attention has been paid in recent years to new configurations for enhancing structural performances [27–30]. There are many researches focusing on the prediction of quasi-static mechanical parameters and explanation of the deformation mechanism. Comparatively, papers on comprehensive study of the dynamic mechanical properties of novel honeycomb structures under axial impact are rather limited.

Above all, the axial crushing of three novel honeycombs, viz. reinforced regular hexagonal honeycomb (RRH-H), tetrachiral cellular honeycomb (TC-H) and bending cellular honeycomb (BC-H), are investigated by both theoretical analysis and numerical crashworthiness optimization design in this paper. Taking the typical type of honeycomb as the research object, theoretical expressions of the mean crushing stress for the three novel honeycombs are derived based on the SSFE theory. Additionally, to evaluate these theoretical solutions for calculating the mean crushing stress, FE simulations of the full-scale elaborator models of honeycombs are conducted through LS-DYNA. Response surface method (RSM) together with multi-objective particle swarm optimization (MOPSO) algorithm is presented to achieve the optimal design under crashworthiness criterion. Finally, for the three optimal structures, the theoretical equations proposed are used to validate the simulation results.

2. Theory

2.1. Calculation of mean crushing stress

The cellular topology charts of three novel honeycomb structures are illustrated in Fig. 1. Considering the symmetry of these structures, a simplification (the shaded area as shown in Fig. 1) is employed to deduce the mean crushing stress. t is foil thickness; L is width of wall.

2.1.1. Theoretical study of RRH-H

The SSFE theory is applied to solve the axial collapse of the honeycomb structures. In the SSFE theory, the mature material has good plasticity that can be regarded as rigid-perfect plastic material. The foil thickness is assumed to be constant and the variation of wavelength $2H$ for different lobes is ignored in this case.

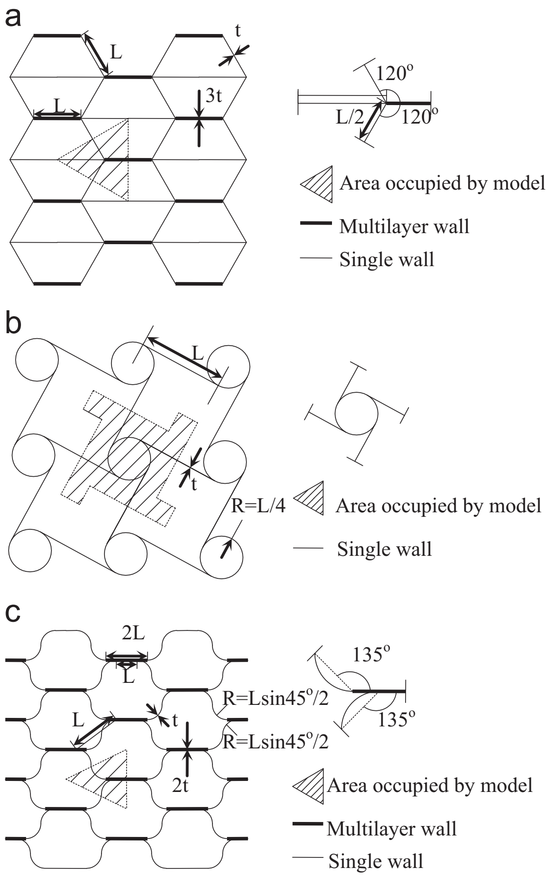

Fig. 1. Geometric configuration of honeycomb: (a) RRH-H; (b) TC-H; and (c) BC-H.

Based on the principle of global equilibrium for shells, the energy absorbed by the trident-shaped honeycomb cell during the axial crushing can be divided into two components: the bending energy due to the formation of the plastic hinges (E_b); the strain energy due to the straining of the membrane (E_m). The external energy work for a complete single fold is equal to the sum of dissipated bending and membrane energy. That is

$$q_c \delta_e 2H = E_b + E_m \tag{1}$$

here q_c is the mean crushing force, δ_e is the effective crushing distance coefficient. In reality, the flange of folding element after deformation is not completely flattened. Therefore, the available crushing distance is smaller than $2H$. As depicted by Wierzbicki and Abramowicz [6,7], the effective compression distance coefficient varied within the range 0.7–0.75. In this case, the value of δ_e is taken as 0.75 for convenience.

The dissipated bending energy can be estimated by summing up the energy dissipation at stationary hinge lines as

$$E_b = \sum_{i=1}^{n} M_0 \theta_i L_i \tag{2}$$

where $M_0 = \sigma_0 t^2/4$ being the plastic bending moment and θ_i is the rotation angle of the ith hinge line, n is the number of hinge lines, and L_i is the length of the ith hinge line. σ_0 is a flow stress of material with power law hardening [31], which may be calculated as

$$\sigma_0 = \sqrt{\sigma_y \sigma_u/(1+n)} \tag{3}$$

here σ_y and σ_u denotes the yield strength and the ultimate strength of the mature material respectively, n is the strain hardening index.