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Nonlinear pull-in instability and free vibration of micro/nanoscale plates with surface energy – A modified couple stress theory model

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ABSTRACT

Effects of surface energy on the pull-in instability and free vibration of electrostatically actuated micro/nanoscale plates are analyzed based on the modified couple stress theory. A reduced-order model is derived to consider the geometrically nonlinear strain, surface energy, the Casimir force and the material length scale simultaneously. Results show that the pull-in voltage and fundamental frequency of the plate are considerably enhanced by the material length scale, surface energy and geometrically nonlinear deformation. However, these quantities are weakened with the inclusion of Casimir force. The effects of surface energy and the material length scale become more significant if the thickness decreases. In addition, the effects of surface energy and geometrically nonlinear strain on the pull-in voltage are the largest for a square plate.

1. Introductions

Many micro/nanodevices, including micro/nano-sensors, switches and actuators, are actuated by electrostatic forces. Due to their advantages of small size, low power consumption, batch production, high isolation [1] and fracture toughness [2–4], they have gained great attention in research. In generally, an electrostatically actuated micro/nano-switch, as shown in Fig. 1, is made up of a conductive deformable electrode (modeled as plate, in this paper) and a fixed electrode. A direct current (DC) voltage applied across the two electrodes makes a bend in the deformable electrode. The deformable electrode collapses to the fixed electrode if the applied DC voltage is beyond a critical value. This phenomenon is the well-known pull-in instability. The corresponding voltage and deflection are denoted as pull-in voltage and pull-in deflection, respectively. Determining the characters of pull-in instability is very important in designing micro/nano-electro-mechanical systems (MEMS/NEMS) and devices. For example, to achieve stable operations and maximize device sensitivity, such instability should be avoided in micro/nano-resonators [5] and micro-mirrors [6]. In switching application [7], the designer used this effect to well control the switch on and off. In all these applications, the information of the pull-in instability must be obtained in order to satisfy the required device specifications.

Based on classical continuum theory, modeling the conductive deformable electrode as a beam [8–13] and plate [1,14–21], the pull-in phenomenon in MEMS devices is studied. However, these models can not explain the size-dependence behaviors of micro/nano-structures. To describe the size-effects of the micro/nano-structures, some modified continuum theories are developed, such as surface elasticity theory, modified couple stress theory and nonlocal elasticity theory.

Due to the large ratio of surface to bulk for nanoscale materials, the effect of surface energy on the mechanical behavior of micro/nano-structures must be considered. Surface elasticity model provided by Gurtin and Murdoch [22] which include the effect of surface energy, has been widely accepted and applied to study the influence of surface effects on mechanical behavior of nanobeams [23–26] and nanoplates [27–30]. For examples, based on the model developed by Wang and Feng [23], He and Lilley [25,26] have investigated the static bending and free vibration of nanobeams incorporating the surface effects. Assadi et al. [28] and Assadi and Farshi [29] studied the influence of surface effect on the vibration behavior of rectangular and circular nanoplates, respectively. Recently, a few works about the pull-in instability of electrostatically actuated nanobeams with consideration of the effect of surface energy are reported [31–34]. For examples, Ma et al. [32] and Koochi et al. [33] studied the effect of surface energy

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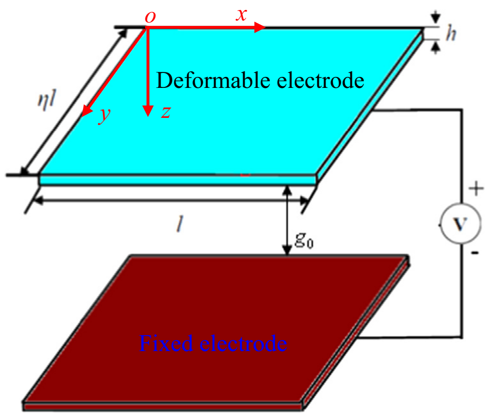

Fig. 1. Sketch of an electrostatically actuated rectangular nanoplate.

on the pull-in instability of an electrostatically actuated cantilever nanobeam. Taking geometric nonlinearity and surface energy into consideration, the pull-in instability of electrostatically actuated nanobeams was discussed in [34]. However, the above mentioned studies [31–34] are limited to one dimensional electrostatically actuated device.

On the other hand, some experiments have demonstrated that the mechanical behaviors of materials are different at different length scales [35–39]. The effects of material length scale constants can be described by couple-stress theory. In this theory, in order to consider the couple stress effects on the deformation energy, two additional material constants are introduced. Yang et al. [40] simplified the theory, and developed a modified couple stress theory which includes only one additional material constant. Recently, the modified couple stress theory provided by Yang et al. [40] is widely used to model the size-dependent behavior of micro-actuators [41–43].

Due to the surface energy describes the surface property and the modified couple stress theory describes the interactions in the bulk, it is natural to consider both of them in studying the size-effects on micro/nanostructures. For example, the effects of surface energy on bending behaviors of nano-beams [44,45] and Kirchhoff nano-plates [46] are studied based on modified couple stress theory. In Ref. [46], it is found that both the microstructure effect in context of modified couple stress theory and surface energy effect are appear if the size reduces to the microscale. They become more significantly with a continuum decrease of the size. Therefore, the combined effects of microstructure and the surface energy should be considered for investing the behaviors of micro-/nano-scale structures.

Unfortunately, there is no study dealt with the combined effects of surface energy and the microstructure effect in the context of modified couple stress theory on electrostatic behavior of micro/nanoscale plates. Micro/nanoscale plates are commonly electrostatically actuated in many MEMS/NEMS devices. Moreover, surface energy and Casimir force have significant effect on electrostatically actuated MEMS/NEMS. Therefore, in this paper, the nonlinear pull-in instability and free vibration of electrostatically actuated micro/nanoscale plates with consideration of surface energy and Casimir force is studied based on modified coupled stress theory. The paper is organized as follows: Section 2 derives the governing equations of the plate subjected to electric and Casimir forces, with consideration of surface energy and von Kármán nonlinear strain, based on modified coupled stress theory. Section 3 describes the reduced-order models for the plate provided in section 2. Method of solving the nonlinear equations for the reduced-order model is outlined in Section 4. Section 5 gives numerical results for the effects of surface energy, geometrically nonlinear deformation, Casimir force and the microstructure effect in the context of modified couple stress theory on the pull-in instability and the fundamental frequency of the plate. Conclusions are drawn in Section 6.

2. Analysis

Different from the classical theory in which the strain energy density depends only on strain, in modified couple stress theory, the strain energy density depends on both strain and gradient of the rotation vector. The strain energy of bulk can be expressed as [40]

$$U=\frac{1}{2}\int_{\Omega}(\sigma_{ij}\varepsilon_{ij}+m_{ij}\chi_{ij})d\Omega \tag{1}$$

where σ_{ij}, m_{ij}, χ_{ij} are the stress, the deviatoric part of couple stress and the symmetric curvature tensor, which are defined as $\sigma_{ij}=C_{ijkl}\varepsilon_{kl}$, $m_{ij}=2Gl_c^2\chi_{ij}$, $\chi_{ij}=(\theta_{i,j}+\theta_{j,i})/2$ and $\theta_i=e_{ijk}u_{k,j}/2$. C_{ijkl} and G are the elasticity module and shear module, respectively. l_c is the material length scale parameter.

As shown in Fig. 1, the deformable electrode is modeled as a plate with length l, width ηl (η is the aspect ratio of the plate, $0<\eta\le 1$) and the height h. The initial gap between the two plates denoted as g_0. The upper surface and the lower surface of the deformable nanoplate are, respectively, denoted by S^+ $(z=h/2)$ and S^- $(z=-h/2)$. The in-plane displacements of the upper plate are expressed as

$$u=u^0-z\frac{\partial w}{\partial x},\ v=v^0-z\frac{\partial w}{\partial y} \tag{2}$$

where u and v are displacements of plate along x-axis and y-axis directions, respectively. u^0, v^0 and w are the displacements of the midplane $(z=0)$ along x-axis, y-axis and z-axis directions, respectively. According to von Kármán plate theory, the strains can be expressed as

$$\varepsilon_x=\varepsilon_x^0-z\frac{\partial^2 w}{\partial x^2},\ \varepsilon_y=\varepsilon_y^0-z\frac{\partial^2 w}{\partial y^2},\text{ and }\varepsilon_{xy}=\varepsilon_{xy}^0-z\frac{\partial^2 w}{\partial x\partial y} \tag{3}$$

where $\varepsilon_x^0=\frac{\partial u_0}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$, $\varepsilon_y^0=\frac{\partial v_0}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$ and $\varepsilon_{xy}^0=\frac{1}{2}\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)$ are the strains of midplane $(z=0)$.

The geometric nonlinearity deformation may also contribute to the rotation gradients. However, it has been confirmed that for geometrically nonlinear plates, the nonlinear part of rotation vector can be neglected, and the rotation vector can be approximately expressed as $\theta_i=e_{ijk}u_{k,j}/2$ [47]. Hence, under considered geometrically nonlinear conditions, the symmetric curvature tensors of the plate are [47–49]

$$\chi_{11}=\frac{\partial^2 w}{\partial x\partial y},\chi_{12}=\chi_{21}=\frac{1}{2}\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x^2}\right),\chi_{13}=\chi_{31}=\frac{1}{4}\left(\frac{\partial^2 v_0}{\partial x^2}-\frac{\partial^2 u_0}{\partial x\partial y}\right),$$
$$\chi_{22}=-\frac{\partial^2 w}{\partial x\partial y},\chi_{23}=\chi_{32}=\frac{1}{4}\left(\frac{\partial^2 v_0}{\partial x\partial y}-\frac{\partial^2 u_0}{\partial y^2}\right),\chi_{33}=0 \tag{4}$$

The strain energy of the bulk includes there parts. One is the potential energy due to bending, the other is the potential energy due to the midplane stretching and the last is the potential energy due to the gradient of the rotation vector. The potential energy due to bending of the plate is expressed as

$$U_1=\frac{1}{2}D\iint(\nabla^2 w)^2-2(1-\upsilon)[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2]dxdy \tag{5}$$

where $D=Eh^3/[12(1-\upsilon^2)]$, E is the Young's modulus and υ is the Poisson's ratio. The potential energy due to the midplane