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Loading and unloading of a power-law hardening spherical contact under stick contact condition

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ABSTRACT

The loading and unloading process of a deformable sphere pressed by a rigid flat is a primary problem in contact mechanics. This work studies the contact of an elastic–plastic sphere with a rigid flat under stick condition during loading and unloading process by the finite element method. The sphere material is assumed isotropic with power-law hardening. The contact load and contact area are calculated at various Poisson's ratios and strain hardening exponents, with the sphere material properties varying from purely elastic to elastic-perfectly plastic. Analytical dimensionless expressions of the contact load, contact area and the residual interference after fully unloading for the power-law hardening materials in the elastic–plastic case are proposed for a wide range of interferences.

1. Introduction

Contact widely exists in many engineering applications [1,2], which notably affects the product performances. Accurate calculation of the contact load and contact area is an important task to help understand the contact behavior and further performances such as friction and wear. However, engineering surfaces are rough in the microscopy scale. When two engineering surfaces are compressed together, the real contact is discontinuous because the contact only occurs at discrete asperities owing to the effect of the surface roughness. What is more, the radii and heights of these asperities are different and distributed randomly. All these characteristics make it difficult to reveal the mechanism of contact between engineering surfaces. For a long time, the contact problem was studied by many researchers for different material structures and in different half spaces. Some researchers focused on the contact of layered materials [3], and some researchers studied the contact in the inhomogeneous half space [4,5]. While in this work, the contact in the homogeneous half space for the solid which was non-layered materials was considered. The original work of Hertz [6] provided a solution for the frictionless, non-adhesive contact of the elastic solids. But actually, real machined surfaces have roughness on a wide range of length scales. Then Greenwood and Williamson [7] pioneered the development of models for contact between complex surfaces. The popular model assumes the asperities having the identical radii and Gaussian distribution heights, and applies Hertz theory to each asperity contact independently. Many scholars followed this model and developed the "statistical approach". The subsequent studies extended the scope to elastic–plastic contact beyond the Hertz restriction, and some elastic–plastic contact models were proposed by Chang et al. [8], Zhao et al. [9], and so on.

In contrast with the statistical approach, the finite element (FE) method is a more accurate means to study the contact, which can also provide the detailed distribution of contact stress and strain. Kogut and Etsion [10] (KE model) gave a precise FE solution for the frictionless elastic–plastic contact of a deformable sphere and a rigid flat with the commercial ANSYS package. They proposed generalized empirical equations to calculate the contact load and contact area, which was not restricted to a specific material or geometry. They found these equations were negligibly affected by the ratios of Young's modulus to yield strength E/Y_0 and those of tangent modulus to Young's modulus E_T/E. Jackson and Green [11] (JG model) provided a similar FE model with much finer meshes. They observed the effect of the deformed geometry on effective hardness and derived some useful expressions for the contact load and contact area as the functions of the interference. This model shows that only at very large interferences, the ratios E/Y_0 will affect the contact behavior. Shankar and Mayuram [12] (SM model) studied the effect of the tangent modulus and the yield strength on the translation behavior of the materials from elastic–plastic to fully plastic case. However, the above FE models [10–12] only took into account the loading case, but the unloading process was all neglected, which is as important as the loading process actually [13–15]. Etsion et al. [16] proposed the first accurate FE solution for the unloading of an elastic–plastic loaded spherical contact. They provided some empirical equations

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Nomenclature

a	contact radius
A	contact area
A_c	critical contact area at yielding inception
A_{max}	maximum contact area before unloading
A^*	dimensionless contact area
A^*_{max}	dimensionless maximum contact area, A_{max}/A_c
E	Young modulus of the sphere
E_T	tangent modulus of the sphere
n	strain hardening exponent
P	contact load
P_c	critical load at yielding inception
P_{max}	maximum contact load
P^*	dimensionless contact load
P^*_{max}	dimensionless maximum contact load, P_{max}/P_c
R	radius of the sphere
w	contact interference
w_c	critical interference at yielding inception
w_{max}	maximum contact interference before unloading
w_{res}	residual contact interference after fully unloading
w^*	dimensionless contact interference
w^*_{max}	dimensionless maximum contact interference, w_{max}/w_c
w^*_{res}	dimensionless residual contact interference after fully unloading
Y_0	virgin yield strength of the sphere
ν	Poisson's ratio of the sphere

for the contact load and contact area versus the contact interference during unloading, and for the residual interference after fully unloading. This model was generalized and independent of the specific materials or radii of the spheres. Kadin et al. [17] improved this model to consider the influence of adhesion on the unloading process. Du et al. [18] researched the loading and unloading process for the gold and ruthenium micro-switches with a similar model.

Most of the aforementioned studies were based on the assumption of frictionless (perfect slip) contact condition. However, friction widely exists in engineering applications such as bolts and brakes. The frictionless assumption may be invalid in dry contact of dissimilar materials as shown experimentally by McGuiggan [19] and Ovcharenko et al. [20]. Therefore, stick condition as a new contact assumption has been applied by researchers. Compared to the frictionless contact condition under which there is no tangential stress in the contact region, the stick condition stops the contact points on the sphere from the relative displacement in the radial direction, but permits the axial displacement of the sphere surface from the rigid flat during unloading [21]. The first analytical solution for the elastic spherical contact problem under stick condition was given by Goodman [22], and many subsequent analyses were performed more exactly. Brizmer et al. [23,24] studied the effect of two contact conditions (perfect slip versus stick) on the loading of elastic and elastic–plastic spherical contact respectively, and derived the dimensionless expressions of the contact load and contact area. They concluded that the interfacial parameters were not so sensitive to the contact condition, independent of the ratios E/Y_0 and E_T/E. But Poisson's ratio had slight influence on the contact load and average pressure, and the evolution of the plastic zone with increasing interference was substantially related to the contact conditions. Ovcharenko et al. [25] performed an experimental investigation to calculate the real contact area between a flat and a sphere during loading and unloading under both perfectly slip and stick condition in the elastic–plastic regime. The experimental results, which were obtained with the copper and the stainless steel spheres pressed against a sapphire flat, were compared with the existing theoretical models and confirmed the validity of the expressions of the contact load and contact area given by Brizmer et al.[23,24]. Zait et al. [26] analyzed the unloading process of an elastic–plastic spherical contact under stick condition for various material properties, and found the obtained load area curve showed a notable difference compared to that obtained under perfect slip condition. Further, they presented the empirical expression for the residual interference considering the effect of Poisson's ratio. The proposed conclusions were in good agreement with experimental results in their paper.

In all the mentioned FE models which investigated the contact of a sphere and a rigid flat, the material of the sphere was generally assumed elastic-isotropic linear hardening with tangent modulus as 2% of the Young's modulus. While in this paper, the contact behavior of the power-law hardening materials is studied. Remberg and Osgood [27] first proposed a direct power relation between stress and plastic strain. Olsson and Larsson [28] researched the force–displacement relations at contact between elastic–plastic adhesive bodies obeying the power hardening law both analytically and numerically during loading and unloading process. Lan and Venkatesh [29] studied the relationship between the hardness and the elastic–plastic properties including the elastic modulus, the yield strength and the strain hardening exponent. Zhao et al. [30] studied the frictionless contact of a power-law hardening elastic–plastic sphere with a rigid flat by the FE method. They presented the dimensionless expressions of the contact load and contact area versus the contact interference during loading and unloading process as the strain hardening exponent varied from 0 to 1. Also, the residual interference after fully unloading at different strain hardening exponent was given. However, the research about the contact between a power hardening elastic–plastic sphere and a rigid flat under stick condition is still missing.

The present work focuses on the contact of a power hardening sphere with a rigid flat during loading and unloading process under stick contact condition by the FE method. The contact load and contact area are calculated at various Poisson's ratios and strain hardening exponents, with the sphere material properties varying from purely elastic to elastic-perfectly plastic. The empirical expressions of the dimensionless contact load, contact area and the residual interference after fully unloading for the power-law hardening materials in the elastic–plastic case are presented for a wide range of interferences.

2. Theoretical background

Fig. 1 shows the loading and unloading process of the contact between a rigid flat and a deformable sphere with a radius R. The dashed lines represent the original profiles of the sphere and the rigid flat before the contact occurs. The solid lines in Fig. 1a show the rigid flat and the sphere with an interference w and a contact radius a related to a contact load P during loading. The solid lines in Fig.1b represent the sphere contour after fully loading with the maximum interference w_{max} and fully unloading with the residual interference w_{res}, and the rigid flat after fully unloading.

Under stick condition of the contact between a rigid flat and a deformable sphere, the critical interference w_c at the yielding inception and the corresponding critical load P_c and critical area A_c are given by Brizmer et al. [23] as follows:

$$wc = \left(C_\nu \frac{\pi(1-\nu^2)}{2}\left(\frac{Y_0}{E}\right)\right)^2 \times R(6.28\nu - 7.83(\nu^2 + 0.0586)) \quad (1)$$