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Size-dependent axial dynamics of magnetically-sensitive strain gradient microbars with end attachments

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A B S T R A C T

The multi-parameter Sturm–Liouville eigenvalue problems, associated with the size-dependent longitudinal vibration of a finite micro-scale bar embedded in orthogonal transverse magnetic fields, are addressed in this study. Derived as a fourth-order partial differential equation, and augmented by enriched higher-order boundary conditions, the mathematical model of the micro-scale bar is predicated on the duo of the strain gradient theory of elasticity and the extended Hamilton's principle. The derived model is tackled with the computation scheme of the power series method. A thorough validation of the simplified form of the model is presented with benchmark results published in the literature. Presented along with the validation study is a comprehensive parametric examination of the influence of the aspect ratio, the material length scale, the mass and stiffness ratios of attachments and the magnetic field strength. Results from the analyses affirm that in the case of a lightweight mass attached to the end of the microbar, the axial resonant frequencies approach that of a microbar with a fixed–free boundary condition. However, in the case of an attachment with a heavy mass the fundamental resonant frequency of the microbar tends to zero. A Pareto analysis of the order of influence of the model's variables unmasks the ratio of the stiffness of the microbar and an attached elastic spring as having the most significant effect on the fundamental axial natural frequency, in the absence of the magnetic field. In the presence of the magnetic field, however, the effects of the end attachments are totally overshadowed by the influence of the magnetic field strength.

1. Introduction

The elementary mathematical models for the longitudinal and torsional vibrations of bars are one-dimensional wave equations, characterized by second order spatial and temporal differential operators. These models have been adroitly used for decades to understand a number of physical phenomena [1]. Typical examples of the application of these elementary models include their use in studying: (i) the behavior of pressure waves in an ideal fluid along the axis of a cylindrical vessel; (ii) the lateral vibrations of taut strings; and (iii) the extensional and torsional behaviors of ultrasonic horns.

Despite the versatile usages of these mathematical models, however, a little bit of uncertainty hovers over their application to certain domains. One such area, where their application appears to have been called into question is in the simulation of high-frequency stress waves in bars [2]. Another notable situation where the elementary model is inadequate is with respect to the testing of low impedance specimens (such as viscoelastic bars). In this respect, a detailed discussion on several approximate theories that have been put forth to deal with the limitations of the model is provided in the study by Anderson [3].

In the current study, we examine another domain of application in which the elementary model of bars is also found to underestimate the actual response of the system it idealizes, namely in relation to the size-dependent vibration response of ultra-thin micro-scale structural element [4]. In this particular instance, the weakness of the elementary model is attributed to its formulation through the classical continuum theory (CCT). Given the underlying assumptions in the CCT, models of solids derived from it only possess length parameters associated with the geometry of such solids [5]. Consequently, the mathematical models derived from it are rendered incapable of predicting size-specific phenomenological traits like dispersion strengthening and the Hall–Petch effect [6]. The phenomenon of size-effect is prominent in micron-scale structures, microstructured porous materials and ductile cellular solids [7–10]. Accumulated experimental evidence for size-specific behavior in micron-scale solids of different materials could be traced to a number of pioneering studies, notable among which are the investigations by

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Kelly [11], Ebeling and Ashby [12], Ashby [13], Hutchinson [14] and Lam and Chong [15].

For the purpose of accommodating size-effect in the prediction of response of micro-scale structures, the presence of material length scale is required in the mathematical models of these structures. The inclusion of the material length scale, in turn, requires the adoption of the axiom of nonlocality in the description of the elastic fields within an infinitesimal bounded volume of the structures [16]. Research studies along this direction have thrown into prominence a new set of higher-order mathematical models of elastic continua containing internal material length scales. Vardoulakis and Sulem [17] chronicled a few of the different formulations of these higher-order theories. Collectively referred to as enriched micro-continuum theories [18], they provide mechanistic routes for assessing the implication of microstructural effects on the behavior of micron-scale or nano-scale structures. Prominent among these theories are the strain gradient theory [19,20], the micropolar and nonlocal elasticity theories [21], the couple stress theory of elasticity [22,23] and the strain gradient theory with surface energy [24]. While some of these theories have been in development for years, their recent rise to prominence is tied to their applicability to the response prediction of nano-sized and micro-sized structures. Small-scale effect in nano-rods have been analyzed, using Eringen's nonlocal elasticity theory, by Aydogdu [25] and Lim et al. [26]. The problem of using the axial vibration behavior of a carbon nanotube for mass sensing applications was addressed by Aydogdu and Filiz [27]. Based on the strain gradient theory with surface energy, the vibration of cytoskeleton, idealized as a network of microtubules, has been studied and reported by Farajour et al. [28] and Civalek et al. [29]. The current investigation is related to the dynamics of microbars, and its foundation is anchored on the strain gradient theory (SGT). As a higher-order theory, the SGT is endowed with additional material length scale parameters in addition to the two classical Lamé constants associated with the theory of linear elastic isotropic materials [30]. The adoption of the SGT for the present work on the axial dynamics of microbars is attributed to the theory's ability to give consistent agreement with empirical observations [31,32].

Microbars are principal components of microelectromechanical systems (MEMs). Thus, the quantification of their dynamic behavior is essential to the optimization of numerous micro-architected MEMs devices such as multi-functional heat exchangers designed with micro-lattices [33], micro-manipulators [34], and micro-turbomachinery [35]. The survey of the literature reveals that studies on the dynamic response of micro-scale bars formulated through the SGT are rather limited, especially when compared to studies on the application of the SGT to the prediction of the response of micro-scale beams and micro-plates. Of the few studies available, Tsepoura et al. [36] applied the SGT to examine the influence of size-effect on the static response of tensioned gradient elastic micro-scale bars. Kahrobaiyan et al. [4] and Akgöz and Civalek [37] studied the longitudinal vibration of microbars. Recently, Güven [38] examined the propagation of longitudinal stress waves in micro-scale bar with the SGT. It is interesting to note that in all of the aforementioned studies, the solution of the higher-order model of the microbar that has been reported is limited to the classical boundary conditions of fixed–fixed ends and fixed–free ends of the microbar.

The modest contribution of this study is in addressing the problem of the effect of end attachments and orthogonal transverse magnetic fields on the longitudinal dynamics of microbars whose higher-order model is derived from the SGT. From the literature survey, to the best of the authors' knowledge, existing studies have not considered these effects, which are crucial for the operation of next generation microsystems [39], on the dynamic or static behavior of microbars. To account for the intrinsic size-dependent trait of the microbar, the study employed two different microstructure-dependent strain energy formalisms to arrive at the eventual models of the microbar. The first strain energy formalism is based on the SGT espoused by Lam et al. [30], while the second originated from the classical work of Mindlin [19]. For the retrieval of the eigenvalues from the derived models, the power series method (PSM) is employed. Through the flexibility and robustness of the PSM, the current study examined, systematically, the shift in the axial resonant frequencies of the microbars when: (i) restrained by a flexible end with a discrete stiffness; (ii) attached to an external discrete mass; and (iii) connected to a flexible end with a discrete stiffness and a discrete mass. Although a major application of the PSM is in dealing with variable-coefficient differential equations, its adoption for the constant-coefficient governing equations derived in the current study is hinged on two reasons. First, the governing equation needs to be supplemented by higher-order boundary terms arising from the effect of the end attachments and the strain gradient effect (as illustrated in Table 1). Due to the nature of these boundary terms, the PSM offers a faster rate of convergence on the eigenvalues of interest as a result of its simplicity in implementation [40]. Second, through the use of the PSM, the retrieval of the eigenvalues is done without having to deal with the transcendental equation that often arises from the exact solution approach. Albeit the PSM in its original form is not easily extended to 2D problems, it forms the basis of methods like the differential transformation method as well as the meshless implementation of the Taylor series method (MITSM) that have been applied to 2D problems [41,42]. Moreover, by supplementing the PSM with the framework of design of experiment, espoused by Montgomery [43], we are able to report on the pattern of the frequency shift precipitated by the size-effect, the end attachments and the presence of the orthogonal transverse magnetic fields. The rest of the presentation is organized as follows. In Section 2, a short introduction to the theoretical foundation of the strain gradient theory is provided. Section 3 demonstrates the application of the PSM to the size-dependent elastodynamics governing equation of microbars with and without attachments. Section 4 is dedicated to the discussion of numerical results, while concluding remarks are detailed in Section 5.

2. Problem foundation

2.1. Kinematic assumptions

It is desired to consider the longitudinal (axial) vibration of a long, narrow micro-scale bar, the schematic of which is shown in Fig. 1. Illustrated in Fig. 2 is the set of boundary conditions of the micro-scale bar to be studied. For this micro-scale bar, the rightward displacement along the length (L) represents a positive motion. The micro-scale bar is constrained to move in a fixed plane to which its initial longitudinal axis (x-axis) belongs. Furthermore, the area of its cross-section is denoted as A, and the undeformed state of the microbar is stress free. In what follows, the particles of the microbar are considered to have a strictly positive density (ρ). For the purpose of modeling, the microbar is assumed to be composed of microelements undergoing micro-deformations. Consequently, the micro-deformation is characterized by considering an infinitesimal volume of the microbar, which is then treated as a differentiable manifold embedded in a Euclidean 3-space $\forall$. The following displacement trial field is taken to hold for the micro-deformation

$$\boldsymbol{u} = u_x \boldsymbol{e_x} + u_y \boldsymbol{e_y} + u_z \boldsymbol{e_z} \tag{1}$$

where u_x, u_y, and u_z represent the components of the trial field in the x, y and z directions of the adopted right-handed Cartesian coordinate system. Since the longitudinal vibration takes place in the axial direction of the microbar, the deformation of its sectional