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Bending of composite plate weakened by square hole

Nirav P Patel [a], Dharmendra S Sharma [b,*]

[a] Mechanical Engineering Department, Institute of Technology, Nirma University, Gandhinagar-Sarkhej Road, Ahmedabad 382481, Gujarat, India
[b] Mechanical Engineering Department, Faculty of Technology and Engineering, The M. S. University of Baroda, Vadodara 390001, Gujarat, India

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ABSTRACT

The present analytic investigations emphasizes on the effect of stacking sequence, ply groups, loading angles, materials and corner radii on the failure strength and moment distribution in symmetric laminated composite plate weakened by a square hole. The generalized close form solution of bending moment around such cut-out is presented by using the Muskhelishvili's complex variable formulation. The layer wise stresses and failure strengths are studied and the failure strengths of laminates are investigated.

1. Introduction

The multilayered plates are being essentially used in industries ranging from space vehicles, aircrafts, biomedical to transportations, house buildings and electronics packaging due to their superiorities like, high strength and light weight. Generally, holes/cut-outs are built into these plates, result into strength degradation. In order to exploit their advantages, the appropriate mathematical tools that deal with their peculiar anisotropic behavior especially when weakened with cutouts, are imperative.

By using various mathematical models, extensive studies have been made on structural analysis of these plates globally and find applications. Goodier [1] and Reissner [2] have studied the moment distribution around circular hole in thin isotropic plate subjected to bending loading. In case of thick isotropic plates, Naghdi [3], and Chen and Archer [4] have presented the solution of stress concentration around circular hole due to bending.

Among all mathematical models, complex variable method [5] is one of the efficient tool to study these plates containing simply or multiply connected domains. By using Muskhelishvili's complex variable approach [5], the problem of stress concentration around various shaped holes for isotropic infinite media subjected to in-plane loading was solved by many researchers like Savin [6] (circular, elliptical, square, and rectangular cut-outs), Lekhnitskii [7] (circular and elliptical cut-outs), Theocaris and Petrou [8] (triangular cut-out), Ukadgaonker and Awasare [9–12] (circular, elliptical, triangular, and rectangular), Rezaeepazhand and Jafari [13] (polygonal cut-out), Batista [14] (polygonal cut-out) and Sharma [15] (polygonal cut-out), etc. The stress distribution is also obtained around circular [7,16], elliptical [16–19], triangular [20–22], square and rectangular [23], and irregular cut-outs [24–26] in composite media subjected to in-plane loading.

The problems of moment distribution around different shaped discontinuities have also been studied. Savin [6] has obtained moment distribution around circular, elliptical, triangular and square hole in infinite isotropic media. Ukadgaonker and Rao [27] have given the general solution for the bending of symmetric laminates based on the formulation of Savin [6] and Lekhnitskii [7].

For the structural analysis of plates, the study of loading conditions, geometry of cuts-outs, material properties is indispensable as the behavior of these plates strongly depend on them [27–29]. The effects of these parameters have been studied around circular [27], polygonal [28], elliptical and triangular hole [29] in isotropic [28] and anisotropic plates [27,29] by using a complex variable method [5]. Hsieh and Hwu [30] used Stroh-like formulation for the solution of bending in the anisotropic case. The solutions of moment distribution around circular, elliptical hole and inclusion were given.

To the best of author's knowledge, only few research articles have reported the moment distribution around regular/irregular holes for anisotropic media. Ukadgaonker and Rao [27], and Sharma and Patel [29] have investigated the effect of stacking sequence on moment distribution around circular [27], elliptical and triangular hole [29] in symmetric laminated plate. The effects of various parameters such as stacking sequence, loading angles,

* Corresponding author. Tel.: +91 9879472897.
E-mail addresses: nirav_npp@yahoo.com (N. Patel), dss_iit@yahoo.com (D. Sharma).

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ply groups, materials and corner radii on the moment distribution and failure strength in case of laminated plates weakened by a square hole are not investigated. In addition to these, the failure strength of composite plate containing a cut-out, under bending loading is also not addressed.

In this study, the values of moments and failure strengths of symmetric laminated plates containing a square hole subjected to bending loading are investigated. The mathematical model of first ply failure theory is used to calculate failure strength of laminate under bending loading. The stresses and Tsai–Hill strengths are described for each layer of laminate. The influence of various parameters such as stacking sequences, ply groups, corner radii, materials and loading angles is investigated.

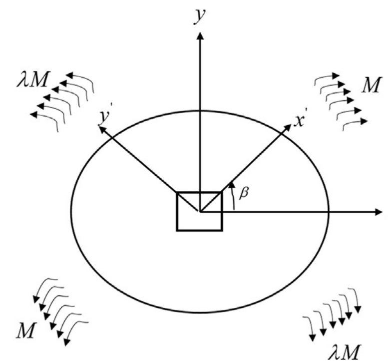

Fig. 1. Plate with square hole.

2. Complex variable formulations

The thin laminated infinite plate of thickness h, made up of N no. of layers, subjected to bending loading, is considered (refer Fig. 1). Each layer is considered to be orthotropic with uniform thickness and perfectly bonded together.

By considering (x, y) as off-axis and $(1, 2)$ as a principal material axis, the transformation equations for kth layer is employed as:

$$\begin{bmatrix} Q_{xx} & Q_{xy} & 2Q_{xs} \\ Q_{yx} & Q_{yy} & 2Q_{ys} \\ Q_{sx} & Q_{sy} & 2Q_{ss} \end{bmatrix}_k = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2-s^2 \end{bmatrix}_k^{-1} \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{12}E_1}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{21}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & \frac{1}{2}G_{12} \end{bmatrix}_k \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2-s^2 \end{bmatrix}_k, \quad (1)$$

where $Q_{ij}(i,j=x,y,s)$ are the stiffness coefficients referred to off-axis $(x-y)$ of kth ply, derived from the Young moduli, $E_i(i=1,2)$, the Poisson ratios, $\nu_{ij}(i,j=1,2)$, the shear moduli, $G_{ij}(i,j=1,2)$, $c=\cos\alpha$ and $s=\sin\alpha$, α is fiber orientation of kth ply.

For the plate subjected to bending, stresses $(\sigma_x, \sigma_y, \tau_{xy})$ in x and y direction can be expressed in-terms of deflection $w(x,y)$ (deflection of the mid-plane in the direction of z-axis) as follows:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = -z \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix}_k \begin{bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{bmatrix}, \quad (2)$$

where $z=$ distance from top plane of a lamina to the mid plane of the laminate and $w_{,ij}(i,j=x,y)$ are the curvatures $(\kappa_1, \kappa_2, \kappa_3)$ of laminate.

The moments in x and y directions, M_x, M_y and M_{xy} (per unit length of the mid plane) can be obtained by taking:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-\frac{t}{2}}^{\frac{t}{2}} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k z\,dz = -\int_{-\frac{t}{2}}^{\frac{t}{2}} \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix}_k \begin{bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{bmatrix} z^2 dz = -\begin{bmatrix} F_{xx} & F_{xy} & F_{xs} \\ F_{yx} & F_{yy} & F_{ys} \\ F_{sx} & F_{sy} & F_{ss} \end{bmatrix} \begin{bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{bmatrix}, \quad (3)$$

where t=thickness of laminate and $F_{ij}(i,j=x,y,s)$ are known as flexural stiffness.

Using Eq. (3) and equilibrium equation, the following 4th order characteristic equation can be obtained [8]:

$$F_{xx}w_{,xxxx} + 4F_{xs}w_{,xxxy} + 2(F_{xy}+2F_{ss})w_{,xxyy} + 4F_{ys}w_{,xyyy} + F_{yy}w_{,yyyy} = 0. \quad (4)$$

The roots of Eq. (4) are

$$\mu_1 = \Upsilon_1 + i\Theta_1, \mu_2 = \Upsilon_2 + i\Theta_2,$$

$$\mu_3 = \overline{\mu_1}, \mu_4 = \overline{\mu_2}. \quad (5)$$

By taking the general equation of deflection in terms of arbitrary and analytic functions, $F_1(\varpi_1)$ and $F_2(\varpi_3)$ as

$$w(x,y) = 2\Re[F_1(\varpi_1) + F_2(\varpi_2)], \quad (6)$$

where $\varpi_j = x + \mu_j y, (j=1,2)$. The values of $\varpi_j (j=1,2)$ can be obtained by transforming the area external to given hole in ϖ-plane to unit circle ζ-plane by generalized transformation function given below:

$$\varpi_j(\zeta) = \frac{R}{2}\left[A_j\left(\zeta^{-1} + \sum_{n=1}^{l} c_n \zeta^{4n-1}\right) + B_j\left(\zeta + \sum_{n=1}^{l} c_n \zeta^{1-4n}\right)\right], (j=1,2), \quad (7)$$

where l=number of terms, $\zeta = e^{i\theta}$,

$$C_n = \frac{-\prod_{p=1}^{n}\{4(p-1)-2\}}{4^n(4n-1)n!},$$

$A_j = 1 + i\mu_j$ and $B_j = 1 - i\mu_j$. The corner radius (r) of the square hole can be related to number of terms (l) by $r=0.1, (l=1)$; $r=0.0417, (l=2)$; $r=0.0240, (l=3)$; $r=0.0161, (l=4)$ and $r=0.0117, (l=5)$.

Now, moments can be calculated by putting the values from Eq. (6) into Eq. (3):

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = -\begin{bmatrix} F_{xx} & F_{xy} & F_{xs} \\ F_{yx} & F_{yy} & F_{ys} \\ F_{sx} & F_{sy} & F_{ss} \end{bmatrix} \begin{bmatrix} 2\Re[\Phi'(\varpi_1) + \Psi'(\varpi_2)] \\ 2\Re[\mu_1^2\Phi'(\varpi_1) + \mu_2^2\Psi'(\varpi_2)] \\ 4\Re[\mu_1\Phi'(\varpi_1) + \mu_2\Psi'(\varpi_2)] \end{bmatrix}, \quad (8)$$

where $\Phi(\varpi_1) = (dF_1/d\varpi_1), \Psi(\varpi_2) = (dF_2/d\varpi_2)$.

Here, $\Phi(\varpi_1)$ and $\Psi(\varpi_2)$ are the stress functions calculated by using method of superposition [5]:

$$\Phi(\varpi_1) = \Phi_1(\varpi_1) + \Phi_2(\varpi_1),$$
$$\Psi(\varpi_2) = \Psi_1(\varpi_2) + \Psi_2(\varpi_2). \quad (9)$$

The stress functions $\Phi_1(\varpi_1)$ and $\Psi_1(\varpi_2)$ can be expressed as follows:

$$\Phi_1(\varpi_1) = \Gamma\varpi_1,$$
$$\Psi_1(\varpi_2) = \Gamma^*\varpi_2, \quad (10)$$

where $\Gamma = B + iC$ and $\Gamma^* = B^* + iC^*$. Here, the rotation is restricted so, $C=0$. Now, the constants, Γ and Γ^* can be calculated by using following equations:

$$\begin{bmatrix} B \\ B^* \\ C^* \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}^{-1} \begin{bmatrix} M_x^\infty \\ M_y^\infty \\ M_{xy}^\infty \end{bmatrix},$$

$$E_{11} = 2(F_{xx} + (L_1^2 - L_2^2)F_{xy} + 2L_1F_{xs}),$$