International Journal of Mechanical Sciences 94-95 (2015) 168–173

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

A parametric model for rotational compliance of a cracked right circular flexure hinge

Xuejun Wang [a], Changli Liu [a,*], Junjie Gu [a], Wenjun Zhang [a,b]

[a] Complex and Intelligent System Research Center, School of Mechanical and Power Engineering, East China University of Science and Technology, Shanghai 200237, China

[b] Department of Mechanical Engineering, University of Saskatchewan, Saskatoon, SK, Canada S7N 5A9

ARTICLE INFO

Article history:
Received 8 August 2014
Received in revised form
16 February 2015
Accepted 18 February 2015
Available online 25 February 2015

Keywords:
Parametric model
Rotational compliance
Right circular flexure hinge
Crack

ABSTRACT

This paper presents the development of a parametric model for the rotational compliance of a cracked right circular flexure hinge. A right circular flexure hinge has been widely used in compliant mechanisms. Particularly in compliant mechanisms, cracks more likely occur in the flexure hinge because it undergoes a periodic deformation. A parametric model has many good uses, as opposed to a numerical model, including fault diagnosis, facility for the real-time control and optimization of design of a compliant mechanism. The rotational compliance of cracked right circular flexure hinge is derived in this paper. This is achieved by the superposition of the rotational compliance of flexure hinge and the compliance change induced by the crack. Without of loss of generality of the result, this paper only considers one type of flexure hinge, namely right circular flexure hinge. The experiment is performed, and the results show that the parametric model is accurate enough for the intended application.

1. Introduction

Compliant mechanisms are mechanical systems that produce the motion along with force through the deformation of the material [1]. Typically, in compliant mechanisms, there is a special structural pattern called flexure hinge (see Fig. 1) which allows for the deformation rather that the “rigid” joint to create apparent displacements or motions. The flexure hinge based compliant mechanism dominates the application of compliant mechanisms according to Cao et al. [2]. It can be imagined from Fig. 1 that the material around a flexure hinge is subject to a cyclic loading, which can however easily generate cracks and then eventually make the whole system fail to function or make the system dysfunction. Detection of cracks in a flexure hinge is thus important to keep the system functioning and resilient [3–5]. Since the right circular flexure hinge enjoys high precision in motion where their center of rotation does not displace as much as other types of flexure hinge, it has been widely used in the applications including mechanical amplifier and manipulator, as discussed by Ouyang et al. [6,7] and Zettl et al. [8]. Hence, only the right circular flexible hinge is considered in this paper without loss of generality.

Many failure symptoms of a compliant mechanism are related to the crack in flexure hinge, and these symptoms are further associated with the stiffness or compliance of flexure hinge. Therefore, to understand the relationship between the crack and stiffness for the flexure hinge is of vital importance to the monitoring of the performance of a compliant mechanism and thus improve its resilience [3,4].

This paper presents a parametric model for the compliance of flexure hinge with an embedded crack on its surface. One benefit of the parametric model with respect to fault diagnosis is such that the characteristics of a crack can be explicitly represented or parameterized that further gains computational efficiency and accuracy in diagnosis [9]. Another benefit is that the parametric model helps a system designer to build a relatively straightforward connection between the crack and compliance such that optimization of a particular design of the compliant mechanism in light of less chance to get cracks (which further means that the mechanism is more reliable) is highly facilitated.

A few models for the compliance of flexure hinge have been proposed especially in the rotational direction (following the right hand rule). These methods are generally classified into three categories: Castigliano's second theorem, inverse formal mapping and, empirical equations through finite element analysis (FEA). Paros et al. [10] introduced an equation by the integration of linear differentials of a beam with circular profile including both full and simplified forms, while the result shows a large discrepancy compared with the

* Corresponding author. Tel.: +86 21 64253538.
E-mail address: clliu@ecust.edu.cn (C. Liu).

http://dx.doi.org/10.1016/j.ijmecsci.2015.02.012
0020-7403/

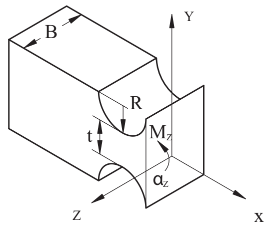

Fig. 1. Circular flexure hinge.

FEA result, and the discrepancy varies with the t/R of circular flexure hinge. After that, more accurate equations were developed by Wu et al. [11] based on the Paros and Weisbord's full equations, in which a novel integral formula was proposed by introducing the so-called centrifugal angle as an integral variable. The result predicted with their model has been found in good agreement with the FEA result. Lobontiu et al. [12] developed a closed-form equation based on Castigliano's second theorem, and a finite element method (FEM) was taken to verify the equation with various geometrical parameters. As a result, the error with their model was less than 10%. Tseytlin [13] figured out a closed-form equation by the so-called inverse conformal mapping technique for a circular approximating contour; the result predicted with their model is much closer to the result predicted with FEA and experimental result than any other parametric equation reported in the literature.

There are many studies on the FEM for modeling of flexural hinge including [14,15], in which dimensionless design charts for three types of flexure elements were developed, and the charts can be used as a design tool to determine the optimal flexure hinge. Yong et al. [16] compared exhaustively existing models for the rotational compliance of flexure hinge with the FEM result as a benchmark for a range of t/R ratios.

Regarding the crack modeling, a number of approaches have been proposed for the beam structure in literatures, and they can be classified into three main categories: discrete spring models, local stiffness reduction, and complex models in two or three dimensions. Dimarogonas [17] and Ostachowicz et al. [18] provided a comprehensive survey of the aforementioned crack modeling approaches. The discrete spring model is one of these approaches. The basic idea of the discrete spring model is to divide a structure into two parts which are pinned at the crack location, and then the crack is considered as a rotational spring on the pin. The shortcoming with this model is that the crack size has not been represented explicitly in the model. The local stiffness reduction method is another crack modeling approach, and the basic idea of this method is to use the equivalent stiffness reduction to model the effect of crack on the structure behavior, and the model can explicitly represent the crack size.

The aforementioned crack modeling methods are mainly discussed in the context of structure health monitoring addressed by Friswell et al. [19]. Papadopoulos et al. [20] used the strain energy release rate (SERR) method, combined with the linear fracture mechanics, to model the local compliance in presence of the crack. In this method, SERR represents the amount of energy per unit length along the crack edge, which is due to the elastic energy in the body and external energy due to loads applied to the system. The SERR further induces stress concentration in the cracked area. The SERR is measured by the stress intensity factor (SIF) [21].

Generally, there are three modes of crack, namely opening, sliding and tearing (Fig. 2) and they correspond to SIFs K_{I} K_{II} and K_{III}. Murakami and Gakkai [22] presented SIFs for several crack situations out of the three basic modes, and the majority of the formulae to compute SIF was numerical instead of analytic due to the difficulty of getting the parametric solution.

At present, to the best of our knowledge, there is no report in the literature on modeling of cracks in flexure hinge, though there are some works about the finite element model of cracked beam-type structure [20], the formula for the SIF of the cracked beam is available but not applicable to the cracked flexure hinge. Therefore, the development of formula to calculate the SIF of the cracked right circular flexure hinge is an interesting work in the field of compliant mechanisms and robots.

The general methodology for deriving the rotational compliance of a cracked flexure hinge in this study is to integrate the rotational compliance of flexure hinge and the compliance change induced due to the crack in flexure hinge. For the rotational compliance, a model is chosen according to [16], and for the compliance change, a new formula is developed based on the SERR method. The rotational compliance of healthy flexure hinge is determined when the geometrical boundaries are given, while the rotational compliance change is derived by having a flexure hinge with a crack be subjected to a moment. As a result, there are two parts of compliances, namely the compliance and compliance change (the two are independent to each other). Thus, the rotational compliance of a cracked right circular flexural hinge can be found by the superposition of these two parts.

The remainder of the paper is organized as follows. Section 2 presents a model for the rotational compliance of a cracked right circular flexure hinge. In Section 3, an experiment is presented to validate the proposed model. A concluding remark is given in Section 4 with a brief discussion of several applications of the parametric model, which are our on-going works.

2. Rotational compliance of a cracked right circular flexure hinge

As mentioned above, the total rotational compliance of a cracked flexure hinge has two parts: the rotational compliance of flexure hinge and the rotational compliance change due to the presence of a crack. In the following discussion, the flexure hinge is subjected to a pure moment only.

2.1. Rotational compliance of the flexure hinge without a crack

Fig. 3 shows the existing models for the rotational compliance of the flexure hinge and their accuracy. In this paper, the right circular flexure hinge is considered, which means that the ratio t/R is 1 (Fig. 3). From Fig. 3, the equation of Schotborgh et al. [15] has the highest accuracy and it is therefore taken for modeling the rotational compliance of the flexure hinge which embeds a crack, that is:

$$C_h = \frac{\alpha_z}{M_z} = \left\{ \frac{EBt^2}{12} \left[-0.0089 + 1.3556\sqrt{\frac{t}{R}} - 0.5227 \left(\sqrt{\frac{t}{2R}} \right)^3 \right] \right\}^{-1} \quad (1)$$

where C_h is the rotational compliance, and E is Young's modulus. It can be seen from the equation that the rotational compliance is associated with the ratio t/R.

2.2. Change in the rotational compliance due to a crack

Fig. 4 describes the geometry of the cracked flexure hinge, where the right side of Fig. 4 is the cross section area of the hinge where the crack lies down.

In this case, the cracked flexure hinge is subjected to a pure moment. According to the relation between SIF and strain energy release rate for the plane stress problem [21], the strain release