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Size-dependent free vibration analysis of electrostatically pre-deformed rectangular micro-plates based on the modified couple stress theory

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ABSTRACT

The modified couple stress theory (MCST) is a non-classical continuum theory which is capable of capturing size-dependent behavior of small structures occurring in micron and sub-micron scales. The objective of the present paper is to investigate the size-dependent free vibration characteristics of rectangular micro-plates pre-deformed by an electric field based on the MCST. To this end, a size-dependent Kirchhoff's plate model is considered and the equation of motion which accounts for the effect of residual and couple stress components as well as the inherent non-linearity of distributed electrostatic excitation is derived using Hamilton's principle. The eigenvalue equation corresponding to the free vibration of electrostatically pre-deformed rectangular micro-plates is also extracted from the equation of motion. This equation is solved numerically using the finite element method (FEM). The results are compared and validated by available analytical and semi-analytical findings for flat micro-plates as well as empirical pull-in observations in the literature and a very good agreement between them is observed. A parametric study is also conducted to investigate the effects of couple stress components as well as electrostatic attraction on both natural frequencies and mode-shapes of fully clamped and simply supported micro-plates. It is found that the size effect on natural frequencies is quite negligible for flat and electrostatically pre-deformed micro-plates when the ratio of plate thickness to the material length scale parameter is larger than 10 and 20, respectively. Furthermore, it is found that couple stress components and electrostatic attraction do not have a sizeable effect on micro-plate mode-shapes.

1. Introduction

Micro-electro-mechanical systems (MEMS) are mostly used as sensors and actuators. Because of their small size, low power consumption and the reliability of their batch fabrications, there are many potential applications in engineering. Resonant micro-sensors as one of the largest categories of MEMS are extensively used in different applications such as signal filtering, and chemical and mass sensing [1,2].

The building blocks of MEMS are electrostatically actuated micro-beams and micro-plates [3]. Besides the major applications of electrostatically actuated micro-beams [4,5], micro-plates-based MEMS can be used in a variety of engineering applications [6]. Due to the high stiffness and simplicity of the production procedure of fully clamped electrostatically actuated micro-plates, they represent major structural components of MEMS devices [7]. In general, an electrostatically actuated micro-plate is an electrically conductive and elastic thin plate suspended over a stationary rigid electrode [8]. In resonant MEMS sensors, the movable electrode is deflected toward the fixed electrode and vibrates about its static deflection, where the direct current (DC) applied voltage is responsible for the static deflection and the oscillatory motion occurs due to the application of the alternating current (AC) voltage [9]. It is noted that, for AC voltage amplitudes much smaller than the DC ones, the micro-plate resonance frequencies are controlled by the DC voltage [10]. Hence, a free vibration analysis of pre-deformed electrostatically actuated micro-plates can provide accurate and promising results for designing MEMS resonators.

The applied DC voltage has an upper limit in which the nonlinear electrostatic attraction overcomes the elastic restoring force of the plate. In this manner, the movable part is suddenly collapsed toward the fixed substrate. This unstable behavior is called pull-in instability, which has simultaneously been observed by Nathanson et al. [11] and Taylor [12]. Also, the upper limit of the DC voltage is called the pull-in voltage.

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Recently, a variety of experiments showed that the material mechanical behavior in small scales is size-dependent [13–16]. Size-dependent behavior is an intrinsic property of certain materials, which emerges when the characteristic size, e.g. the diameter or the thickness, is comparable to the material length scale parameter. Material length scale parameter for a specific material can be determined using some typical experiments such as micro-torsion test [13], micro-bend test [14,15] and micro/nano-indentation test [16–18]. For example, the length scale parameter for single crystals of Al, Ag, Ni, polycrystalline Cu, ploy-synthetically twinned (PST) lamellar α_2-TiAl and $\gamma-$TiAl have been determined, respectively, as 2762 nm, 6233 nm, 4315 nm, 1120 nm, 74 nm and 49 nm [19]. Also the length scale parameter for silicon, as the frequently used material in MEMS resonators, was presented by Rahaeifard et al. [20] as $l=592$ nm.

The classical continuum mechanics cannot predict the size-dependent behavior of materials which occurs in micron and sub-micron scale structures. To remove this incapability of classical continuum mechanics, the size-dependent continuum theories have been developed [13,21–26]. These theories include some additional material constants besides two classical Lame's constants for isotropic materials: the classical couple stress theory (CCST) [22], the classical strain gradient theory (CSGT) [13], the modified couple stress theory (MCST) [25] and the strain gradient theory (SGT) [26] which include two, five, one and three additional material constants, respectively.

In view of the difficulties involved in determining higher-order material constants [26,27], the MCST of elasticity has been elaborated by Yang et al. [25] which has very desirable features such as having only one additional material length scale parameter and using a symmetric couple stress tensor. It should be noted that the difference between the results of this theory and those obtained by the other higher-order theories such as the SGT for problems which investigate the bending of thin micro-structures is negligible [28]. Hence, this theory can be successfully utilized to predict the characteristics of MEMS resonators.

The size-dependent behavior of mechanical structures at micron and sub-micron scales motivated many researchers to develop some mechanical models using the size-dependent theories. Although many researchers have dealt with the size-dependent behavior of micro-beams-based structures [29–31], the research efforts devoted to micro-plates are very limited. Here the studies undertaken on the size-dependent micro-plate models are reviewed. Tsiatas [32] presented a new Kirchhoff's plate model based on the MCST, which can predict the size-dependent behavior of micro-plates with arbitrary shapes. He solved the governing equation of motion using the method of fundamental solutions (MFS) and investigated the size effect on the static deflection of the micro-plate. He showed that accounting for the effect of couple stress components increases the bending rigidity of the plate. Also, he found that increasing the material length scale parameter decreases the static deflection of the plate. Furthermore, this behavior is totally independent of the plate aspect ratio and boundary conditions. Asghari [33] developed a geometric non-linear and size-dependent Kirchhoff's plate model based on the MCST. He utilized Hamilton's principle to derive the governing equations of motion and the corresponding boundary conditions for micro-plates with arbitrary shapes. Jomehzadeh et al. [34] investigated the effect of size on the natural frequencies of circular and rectangular Kirchhoff's micro-plates with two simply supported opposite edges using the MCST. They found that accounting for the effect of couple stress components increases both bending rigidity and natural frequencies of the micro-plate. Akgöz and Civalek [35] studied bending, buckling and free vibration of simply supported rectangular Kirchhoff's micro-plates resting on an elastic medium using the MCST. Ke et al. [36] studied the effect of couple stress components on the natural frequencies of rectangular Mindlin's micro-plates using the p-version of Ritz's method. They showed that the natural frequencies of the plate significantly increase when the plate thickness becomes comparable to the material length scale parameter. Roque et al. [37] investigated the static bending of rectangular Mindlin's micro-plates using the MCST. They solved the governing equations using a meshless method based on collocation with radial basis functions. The increase in the bending rigidity of the micro-plate due to the couple stress effects was also reported in this study. Zhang et al. [38] proposed a non-classical rectangular Mindlin's plate element with four nodes and 15-DOF (degrees of freedom) per node based on the MCST, which satisfies the C_1 weak continuity conditions. They studied static bending, buckling and free vibrations using their proposed element, which is free of shear locking. Ke et al. [39] studied static bending, buckling and free vibration of functionally graded annular Mindlin's micro-plates based on the MCST. They employed the differential quadrature method (DQM) to solve the governing equations. They showed that the effect of couple stress components can be neglected for micro-plates in which the ratio of the plate thickness to the material length scale parameter becomes larger than 10. Wang et al. [40] presented an algorithm for the asymmetrically non-linear size-dependent free vibration analysis of a circular micro-plate based on the MCST. They reduced the governing partial differential equation (PDE) of motion to corresponding ordinary ones by eliminating the time variable through Kantorovich's method following an assumed simple harmonic function in time. The resulting non-linear spatial boundary value problem was then solved numerically using the shooting method. Through this procedure, they investigated the combined effects of large amplitudes and size-dependency on free-vibration characteristics of asymmetrically circular micro-plates. Thai and Choi [41] developed size-dependent linear and non-linear functionally graded Kirchhoff's and Mindlin's rectangular plate models based on the MCST. They also studied the effect of couple stress components on static bending, buckling and free vibration behaviors analytically for simply supported plates. Similar investigations for size-dependent Reddy's micro-plates are also presented by Thai and Kim [42] based on MCST.

Although size-dependent analysis of electrically actuated micro-beams has been carried out in some researches to date, the research efforts devoted to micro-plates are very limited. Herein the most pioneering works undertaken on the size-dependent pull-in analysis of micro-structures are reviewed. Rahaeifard et al. [20] presented a size-dependent electro-mechanical model for pull-in analysis of electrically actuated micro-cantilevers based on the MCST. They could remove the existing gap between the empirical observations and the classical results using their model. They also studied the size-effect on pull-in instability of geometric non-linear micro-bridges using the same theory [43]. Kong [44] introduced an analytical approximate solution to static pull-in problem and calculated pull-in voltage and pull-in displacement based on the MCST using the Rayleigh–Ritz method. He found that pull-in voltage predicted by the MCST is 3.1 times greater than that predicted by the CT when the micro-beam thickness is equal to material length scale parameter. Furthermore, the normalized pull-in displacement is size-independent and equal to 0.448 and 0.398 for cantilever and clamped–clamped micro-beams, respectively. Ghayesh et al. [45] studied the non-linear size-dependent behavior of electrically actuated MEMS resonators subjected to combined AC and DC voltages. They applied the high-dimension Galerkin-based reduced order model together with the pseudo-arclength continuation technique on their continuous beam model to extract the frequency and force responses of the system as well as its time histories and phase portraits under both primary and secondary