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A spectral collocation solution for in-plane eigenvalue analysis of skew plates

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ABSTRACT

Free in-plane vibration analysis of isotropic plates with skew geometry is carried out using the spectral collocation method. The mathematical formulation of the discretized spectral solution is expressed in a concise matrix form which can be directly and easily coded in modern mathematical software packages. A rather comprehensive set of plate cases with various skew angles, aspect ratios, and boundary conditions is presented, with the aim of both showing the rate of convergence and degree of accuracy of the adopted method and providing useful design guidelines related to the effect of the plate geometrical parameters on the fundamental in-plane frequency value.

1. Introduction

Accurate computation of the in-plane modal characteristics of plates can be of utmost importance in some engineering applications such as transmission of high frequency vibration through a built-up structure [1,2] or excitation of thin plates subjected to high speed tangential flows.

Owing to the practical interest of the problem, some researchers investigated the in-plane vibration of plates according to different mathematical approaches. About two decades ago, Bardell et al. [3] used the Ritz method to study the in-plane frequencies and mode shapes of isotropic rectangular plates with various boundary conditions. The same method was adopted later by others to study the effect of ply orientation in orthotropic and laminated plates [4,5], the influence of nonuniform elastically restrained boundaries [6] and the modal properties when the plate has non-rectangular geometry [7]. Gorman [8–10] applied the superposition method to accurately predict the in-plane frequencies of rectangular plates with fully free and clamped edges and uniform elastic supports normal to the boundary. The Kantorovich–Krylov method was employed by Wang and Wereley [11] to compute free in-plane vibration characteristics of rectangular isotropic plates with various combinations of clamped and free edges. Finally, a series of solution is obtained in Refs. [12–14] for the in-plane vibration analysis of isotropic and orthotropic plates with elastically restrained boundaries.

Besides the aforementioned approximate analytical-type and numerical studies, some exact solutions of the free in-plane vibrations of rectangular plates are also available in the open literature. Gorman [15] analyzed plates having at least two opposite edges simply supported and the other edges free or clamped. In Gorman's work, two distinct types of simple support boundary conditions are formulated: so called simple support type 1 (SS1), where the normal stress and tangential displacement along the edge are zeros, and simple support type 2 (SS2), where normal displacement and tangential stress are zeros. Xing and Liu's work [16] is another significant contribution in this field. They employed the separation of variables' method to obtain exact solutions of natural frequencies and mode shapes when at least two opposite edges had either types of simple support conditions previously introduced. All possible exact solutions were successfully obtained, including cases which were not available before. An extension of the same exact procedure to orthotropic plates is presented in Ref. [17].

Despite the availability of the works cited above, the amount of research devoted to free in-plane vibration of plates is still extremely small in comparison to that devoted to free transverse vibration of plates. In particular, very little attention has been given to in-plane vibration analysis of non-rectangular plates with straight edges. To the best of author's knowledge, the topic is discussed only in the work by Singh et al. [7], where a modified form of the Rayleigh–Ritz method is adopted to study rhombic plates. Only few cases with clamped and free edges are numerically investigated in Ref. [7]. Skew plates are of practical interest in the aerospace industry due to the increasing use of such components in aircraft and space vehicles. Indeed, there are extensive studies on the transverse vibration of plates with skew geometry (see, for instance, [18,19] and references therein). The same cannot be said for the in-plane vibration problem. The main purpose

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of the present paper is to contribute to the literature on vibration of skew plates by providing complete sets of vibration data related to in-plane modal properties of plates with various boundary conditions, different aspect ratios and small and large skew angles.

Since exact solutions cannot be obtained for the problem under study, a numerical approach must be employed. Instead of using a classical finite element method (FEM), the solution of the free in-plane vibration of skew plates is obtained here by a spectral collocation method. FEM typically requires both huge computational resource to accurately capture high frequency modal behavior and remeshing for any variation of geometrical parameters. Therefore, it appears to be an unsuitable choice especially when extensive optimization and parametric analysis are to be performed.

Spectral methods [20] are known to have high rate of convergence and accuracy. There are various kinds of spectral methods, which can be classified according to the selection of basis and weighting functions in the numerical procedure. The spectral collocation method used here, also known as the Chebyshev collocation method or the pseudospectral method [21], can be considered as a global spectral method that performs a collocation process, i.e., weighting functions are delta functions centered at special grid points called collocation points. Since the mathematical formulation is simple and powerful enough to produce approximate solutions close to exact values, this method has been largely adopted with success in solving partial differential equations governing many physical phenomena such as fluid dynamics and wave motion. It was also used for the solution of structural mechanics problems. Lin and Jen [22] used the pseudospectral method for computing the bending response of laminated anisotropic plates. The eigenvalues analysis of Timoshenko beams and axisymmetric Mindlin plates is presented by Lee and Schultz [23]. More recently, Sari and Butcher applied the pseudospectral method to study the effect of damaged boundaries on the free transverse vibration of thin, moderately thick and thick rectangular plates [24–26]. It is worth mentioning that, as pointed out by Shu [27], the pseudospectral method is identical to the differential quadrature method (DQM) [28] when the grid points of DQM are chosen to be the Chebyshev collocation points. It is also noted that, differently from the out-of-plane free vibration problem of thin plates, the application of DQM to in-plane vibration analysis of plates is easier since difficulty in dealing with multiple boundary conditions [29,30] does not exist.

The paper is organized as follows. Section 2 presents the mathematical formulation in terms of equations of motion and boundary conditions of the problem under study, and the related discretization procedure and eigenvalue problem. The discretization of the boundary-value problem is obtained in a concise matrix form which can be directly and easily coded in modern mathematical software packages. Some numerical results are shown in Section 3. First, the rate of convergence of the method is discussed with respect to FEM solutions and for varying skew angles and boundary conditions. Then, the accuracy of the present approach is evaluated by comparison with some reference cases available in the literature. Finally, the fundamental in-plane frequencies of plates with various aspect ratios, skew angles and boundary conditions are reported. Section 4 contains some concluding remarks.

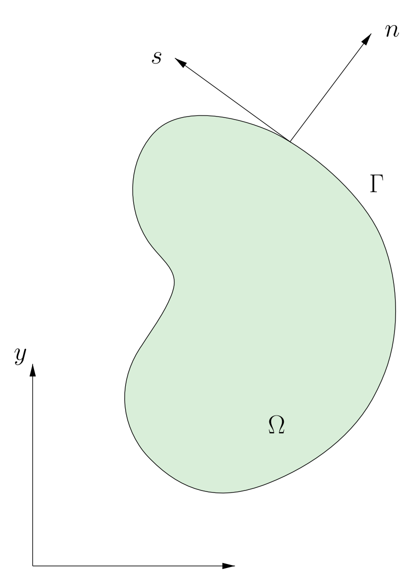

Fig. 1. A plate of generic shape Ω and boundary Γ.

2. Mathematical formulation

2.1. In-plane equations of motion and boundary conditions

Under the small strains assumption, the in-plane dynamic equilibrium of a homogeneous isotropic plate of thickness h with undeformed midplane Ω (see Fig. 1) can be expressed in weak form through the principle of virtual displacements as follows:

$$\int_\Omega \left[\delta\frac{\partial u}{\partial x}N_{xx}+\delta\frac{\partial v}{\partial y}N_{yy}+\left(\delta\frac{\partial u}{\partial y}+\delta\frac{\partial v}{\partial x}\right)N_{xy}\right]\mathrm{d}y\mathrm{d}x = -\int_\Omega\left[\delta u\, m\frac{\partial^2 u}{\partial t^2}+\delta v\, m\frac{\partial^2 v}{\partial t^2}\right]\mathrm{d}y\mathrm{d}x \tag{1}$$

where $u=u(x,y,t)$ and $v=v(x,y,t)$ are the displacement components along the in-plane (x,y) cartesian coordinate directions, δ denotes the virtual variation, $m=\rho h$ is the mass per unit area and $N_{\alpha\beta}$ $(\alpha, \beta = x, y)$ are the in-plane stress resultants.

After integrating by parts (Eq. (1)), the equilibrium can be written as

$$\int_\Gamma\left[\delta u\, N_{xx}n_x+\delta v\, N_{yy}n_y+\delta u\, N_{xy}n_y+\delta v\, N_{xy}n_x\right]\mathrm{d}s - \int_\Omega\left[\delta u\frac{\partial N_{xx}}{\partial x}+\delta v\frac{\partial N_{yy}}{\partial y}+\delta u\frac{\partial N_{xy}}{\partial y}+\delta v\frac{\partial N_{xy}}{\partial x}\right]\mathrm{d}y\mathrm{d}x = -\int_\Omega\left[\delta u\, m\frac{\partial^2 u}{\partial t^2}+\delta v\, m\frac{\partial^2 v}{\partial t^2}\right]\mathrm{d}y\mathrm{d}x \tag{2}$$

where Γ is the plate boundary and n_x and n_y are the components of the outward normal $\boldsymbol{n}$ at a point on Γ. Making use of the constitutive equations and exploiting the arbitrariness of the virtual variations over Ω, the in-plane equations of motion can be written in matrix form as

$$\begin{bmatrix}\mathcal{L}_{11} & \mathcal{L}_{12}\\ \mathcal{L}_{21} & \mathcal{L}_{22}\end{bmatrix}\begin{Bmatrix}u\\ v\end{Bmatrix} = m\frac{\partial^2}{\partial t^2}\begin{Bmatrix}u\\ v\end{Bmatrix} \tag{3}$$

where the elements of the 2×2 matrix of linear differential operators are given by

$$\mathcal{L}_{11}=A_{11}\frac{\partial^2}{\partial x^2}+A_{66}\frac{\partial^2}{\partial y^2},\quad \mathcal{L}_{12}=(A_{12}+A_{66})\frac{\partial^2}{\partial x\partial y}$$
$$\mathcal{L}_{21}=\mathcal{L}_{12},\quad \mathcal{L}_{22}=A_{66}\frac{\partial^2}{\partial x^2}+A_{22}\frac{\partial^2}{\partial y^2} \tag{4}$$

The quantities A_{ij} are the in-plane rigidities of the plate defined as $A_{11}=Eh/(1-\nu^2)$, $A_{12}=\nu A_{11}$, $A_{22}=A_{11}$, and $A_{66}=Eh/[2(1+\nu)]$, where E is Young's modulus and ν is Poisson's ratio.

The boundary integral in Eq. (2) can be alternatively written as

$$\int_\Gamma\left[\delta u_n N_{nn}+\delta u_s N_{ns}\right]\mathrm{d}s \tag{5}$$

where u_n and u_s are the boundary displacements along the normal and tangential directions, respectively, on the boundary Γ (see Fig. 1), and N_{nn} and N_{ns} are the corresponding boundary stress resultants.