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Transverse vibration of isotropic thick rectangular plates based on new inverse trigonometric shear deformation theories

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ABSTRACT

The present investigation deals with the proposition of four new inverse trigonometric shear deformation theories to study free vibration characteristics of isotropic thick rectangular plates subjected to various boundary conditions. The proposed theories exactly satisfy the transverse stress boundary conditions on the bottom and top surfaces of the plate, which were true in earlier shear deformation theories also. There are various methods to handle the said type of problems, but the Rayleigh–Ritz method is found to be efficient because it can very well handle all types of classical boundary conditions. Accordingly, the trial functions denoting the displacement components are expressed as the linear combinations of simple algebraic polynomials. The main objective of this investigation is to study the effects of geometric configurations and deformation theories on the frequency parameters as these may be important for design engineers. The new results for frequency parameters are incorporated after performing a test of convergence and validation in special cases with the available literature. 3-D mode shapes are also plotted to show the deflected shapes of the plate.

1. Introduction

In general, there occurs negligible variation of material properties in case of isotropic structural members (beams, plates, etc.) even with the variation of geometric configurations. As present study is concerned with the vibration behavior of plate elements, the literature survey includes the varieties of works related to the plate elements (thin, thick, laminated, composite and sandwich) based on analytical as well as computational techniques. Exhaustive studies concerned with these behavior can be found in [1–5]. Especially, different mechanical response (static, vibration, buckling, etc.) of the plate elements based on specific shear deformation theories are discussed in these literature. Authors throughout the globe mainly implement previously proposed theories for such studies, except for a few. In this regard, an excellent communication is that of Leissa [6] who has used Ritz method to find analytical results for the free vibration of thin rectangular plates. A two-dimensional plate theory (2-DPT) was given by Reissner [7] to the problem of transverse bending of homogeneous plates. Reddy [8] has proposed a higher-order nonlinear shear deformation theory to obtain exact solutions of simply supported plates. Again in another article, Reddy and Phan [9] have used this theory to the elastic plates to determine the natural frequencies and buckling loads. Exact closed-form solutions of symmetric cross-ply laminates were investigated in [10] using the first-order shear deformation theory. Two C^0 assumed finite element formulations of Reddy's higher-order theory were used by Nayak et al. [11] to obtain the natural frequencies of composite and sandwich plates. Hosseini-Hashemi and Arsanjani [12] have presented the exact closed form characteristic equations for the Mindlin plates with two opposite edges simply supported. Shimpi and Patel [13] have implemented two-variable refined plate theory for free vibration analysis of plates. Two new displacement based first-order shear deformation plate theories were introduced by Shimpi et al. [14] to study the dynamic problems. Aydogdu [15] has proposed a new

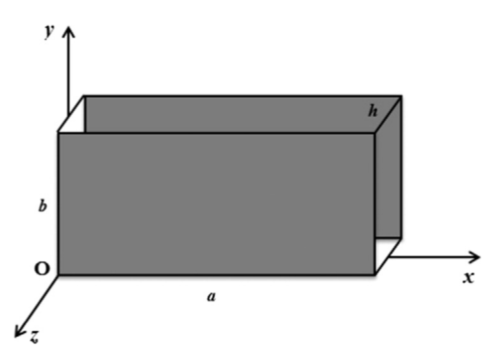

Fig. 1. A typical isotropic rectangular plate with the cartesian coordinates.

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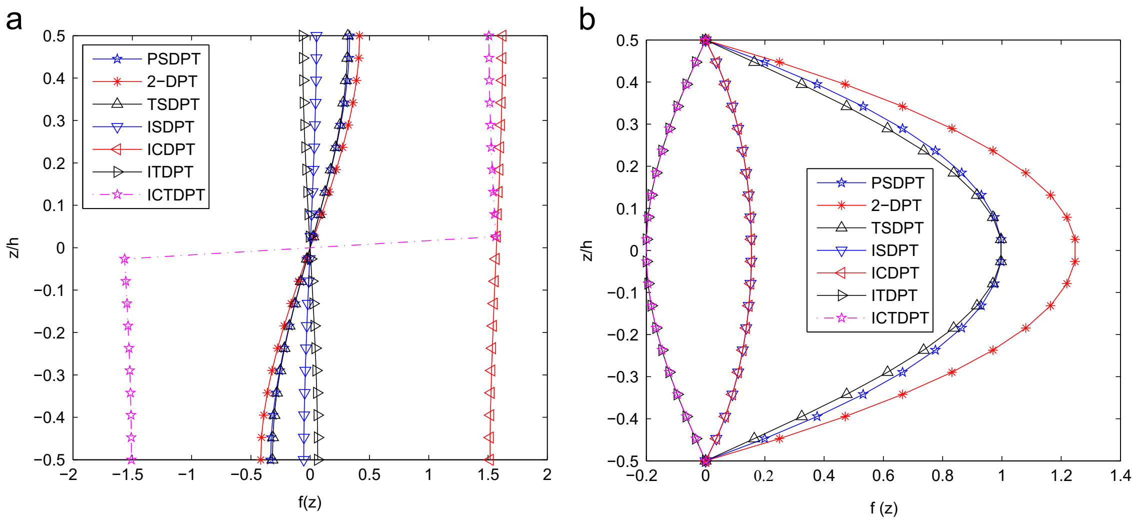

Fig. 2. Behavior of (a) shape functions and (b) derivatives of shape functions along thickness of isotropic rectangular plate.

higher order shear deformation laminated composite plate theory from 3-D elasticity solutions by using an inverse method. Various shear deformation theories have been implemented by Xiang et al. [16] for the modelling of isotropic, sandwich and laminated plates. Flexural vibration of Lévy-type rectangular plates has been studied by Hosseini-Hashemi et al. [17] within the framework of third-order shear deformationtheory. A new inverse tangent shear deformation theory (ITSDT) was presented by Thai et al. [18] for the static, free vibration and buckling analysis of laminated composite and sandwich plates. Jin et al. [19] have presented a new three-dimensional exact solution for free vibrations of arbitrarily thick functionally graded rectangular plates. Three-dimensional solution procedures are also given to study free vibration analysis of isotropic and orthotropic conical shells with elastic boundary restraints in [20], of thick functionally graded conical, cylindrical shell and annular plate structures with arbitrary elastic restraints in [21] and of thick cylindrical shells with general end conditions and resting on elastic foundations in [22]. A unified modified Fourier solution based on the first order shear deformation theory is developed by Jin et al. [23] for vibrations of various composite laminated structure elements of revolution with general elastic restraints including cylindrical, conical, spherical shells and annular plates. In this regard, Ye et al. [24] have used modified Fourier solution based on first-order shear deformation theory for the free vibration problems of moderately thick composite laminated plates with general boundary restraints and internal line supports.

As present study is associated with free vibration of isotropic plates by means of Rayleigh–Ritz method, we have also discussed a few existing works related to this computational procedure. The Rayleigh–Ritz method (after Lord Rayleigh and Walther Ritz) is an efficient numerical technique for the analysis of structural members, which may handle any possible sets of boundary conditions easily. Dawe and Roufaeil [25] have implemented the Rayleigh–Ritz method to predict the natural frequencies of flexural vibration of square plates having general boundary conditions based on Mindlin plate theory. Deobald and Gibson [26] have used Rayleigh–Ritz technique to model vibration characteristics of rectangular orthotropic plates. Transverse vibration of elliptic and circular plates using orthogonal polynomials in Rayleigh–Ritz method has been investigated by Singh and Chakraverty [27–29] for different boundary conditions viz. completely free, simply

Table 1
Shear deformation plate theories and their respective coefficients: C_1, C_2 and C_3.

Source	Theory	Coefficients	Expression
Reddy [8,10]	PSDPT	C_1	$\int_{-1/2}^{1/2}\{\bar{z}(1-4\bar{z}^2/3)\}^2\,d\bar{z}$
		C_2	$\int_{-1/2}^{1/2}\{\bar{z}^2(1-4\bar{z}^2/3)\}\,d\bar{z}$
		C_3	$\int_{-1/2}^{1/2}(1-11\bar{z}^2/3)\,d\bar{z}$
Reissner [7]	2-DPT	C_1	$\int_{-1/2}^{1/2}\left\{\frac{5\bar{z}}{4}\left(1-\frac{4\bar{z}^2}{3}\right)\right\}^2 d\bar{z}$
		C_2	$\int_{-1/2}^{1/2}\frac{5\bar{z}^2}{4}\left(1-\frac{4\bar{z}^2}{3}\right) d\bar{z}$
		C_3	$\int_{-1/2}^{1/2}\left(\frac{5}{4}-\frac{15\bar{z}^2}{3}\right)^2 d\bar{z}$
Touratier [34]	TSDPT	C_1	$(1/\pi^2)\int_{-1/2}^{1/2}\sin^2(\pi\bar{z})\,d\bar{z}$
		C_2	$(1/\pi)\int_{-1/2}^{1/2}\bar{z}\sin(\pi\bar{z})\,d\bar{z}$
		C_3	$\int_{-1/2}^{1/2}\cos^2(\pi\bar{z})\,d\bar{z}$
Proposed	ISDPT	C_1	$\int_{-1/2}^{1/2}\left(\frac{2\bar{z}}{\sqrt{3}}-\sin^{-1}\bar{z}\right)^2 d\bar{z}$
		C_2	$\int_{-1/2}^{1/2}\left(\frac{2\bar{z}^2}{\sqrt{3}}-\bar{z}\sin^{-1}\bar{z}\right) d\bar{z}$
		C_3	$\int_{-1/2}^{1/2}\left(\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{1-\bar{z}^2}}\right)^2 d\bar{z}$
Proposed	ICDPT	C_1	$\int_{-1/2}^{1/2}\left(\frac{2\bar{z}}{\sqrt{3}}+\cos^{-1}\bar{z}\right)^2 d\bar{z}$
		C_2	$\int_{-1/2}^{1/2}\left(\frac{2\bar{z}^2}{\sqrt{3}}+\bar{z}\cos^{-1}\bar{z}\right) d\bar{z}$
		C_3	$\int_{-1/2}^{1/2}\left(\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{1-\bar{z}^2}}\right)^2 d\bar{z}$
Proposed	ITDPT	C_1	$\int_{-1/2}^{1/2}\left(\frac{4\bar{z}}{5}-\tan^{-1}\bar{z}\right)^2 d\bar{z}$
		C_2	$\int_{-1/2}^{1/2}\left(\frac{4\bar{z}^2}{5}-\bar{z}\tan^{-1}\bar{z}\right) d\bar{z}$
		C_3	$\int_{-1/2}^{1/2}\left(\frac{4}{5}-\frac{1}{1+\bar{z}^2}\right)^2 d\bar{z}$
Proposed	ICTDPT	C_1	$\int_{-1/2}^{1/2}\left(\frac{4\bar{z}}{5}+\cot^{-1}\bar{z}\right)^2 d\bar{z}$
		C_2	$\int_{-1/2}^{1/2}\left(\frac{4\bar{z}^2}{5}+\bar{z}\cot^{-1}\bar{z}\right) d\bar{z}$
		C_3	$\int_{-1/2}^{1/2}\left(\frac{4}{5}-\frac{1}{1+\bar{z}^2}\right)^2 d\bar{z}$