International Journal of Mechanical Sciences 94-95 (2015) 232–243

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

A nonlocal finite element method for torsional statics and dynamics of circular nanostructures

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ARTICLE INFO

Article history:
Received 12 May 2014
Received in revised form
20 January 2015
Accepted 5 March 2015
Available online 14 March 2015

Keywords:
Nanoscale
NL-FEM
Nonlocal elasticity
Variational principle
Torsion

ABSTRACT

The torsional static and dynamic nonlocal effects for circular nanostructures subjected to concentrated and distributed torques are investigated based on the nonlocal elasticity stress theory. The total strain energy and kinetic energy components are derived and the variational energy principle is applied to derive the governing equation of motion and the corresponding boundary conditions. A new nonlocal finite element method (NL-FEM) is developed to solve the integral nonlocal equation. New numerical solutions for statics and dynamics of nonlocal nanoshafts, nanorods and nanotubes with various loads and boundary conditions are presented. The NL-FE numerical solutions are compared with analytical solutions obtained by solving the differential nonlocal equation. It is observed that the deformation angle as well as the ratio of nonlocal to classical deformation angle increases with increasing nonlocal nanoscale while the natural frequency for free torsional vibration decreases with increasing nanoscale. This paper concludes that the analytical nonlocal model and solutions, which apply the differential nonlocal constitutive relation, fails to capture the nonlocal boundary effects. The NL-FEM, which solves directly the original integral nonlocal stress relation, demonstrates nonlocal boundary effects for all cases of study. The differences of the differential and integral nonlocal stress relations are reported using representative numerical examples.

1. Introduction

The present state of development for new technologies using advanced materials and structures is moving towards a minute length scale (i.e. micro- or nano-scale) and this is becoming the root or progress in nanotechnology. When the size of a body or a structure enters the micro- and nano-ranges, the material exhibits specific and interesting nonclassical mechanical, chemical, electrical properties, etc. The classical continuum theories and models are unable to depict the influence due to this minute-length scale [1,2]. Therefore, new continuum mechanics models or molecular/atomic dynamic simulation approaches are thus required. Nonlocal elasticity theory is one of the continuum models which was first developed by Eringen [3–5] and his associates [6]. It is based on the assumption that the stress field at a point in an elastic continuum not only depends on the strain at that point but also depends on strains at all other points in the body.

Since the discovery of carbon nanotubes (CNTs) by Iijima [7] in 1991, nanostructures are being increasingly used because of their exceptional mechanical properties such as large Young's modulus [8], flexibility [9], high strain and stability [10,11] and conductivity properties [12]. In recent years, a number of studies on nonlocal fields based on the nonlocal constitutive relation of Eringen [3–5] have been reported. Most of the studies focused on various issues of practical interest for carbon nanotubes [13–21], nanobeams [16,19,22–29], nanorods [30–33], nanoplate/graphane sheets [14,34], etc. The main research interests include either static behavior such as bending [19,22,25,27,28,29,35], buckling [15,18,21,22,25,27,28,33,35,36] or the dynamic behavior such as vibration [13,22,25,27,28,30,37,38,39], and wave propagation [16,20], etc.

In particular, Sudak [36] presented column buckling of multi-walled carbon nanotubes (MWCNTs) based on the nonlocal continuum mechanics model; Wang et al. [21] also used this model to investigate torsional buckling of moderately large carbon nanotubes (CNTs) with different aspect ratios. This nonlocal continuum mechanics model was further implemented by several researchers to study torsional vibration of nanostructures i.e. CNTs [38], nanorods [31] and dynamics [40,41]. Narender et al. [32] also used a strain gradient model to study torsional vibration of micro- and nano-rods. Shell models were also applied to study torsional buckling of CNTs

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http://dx.doi.org/10.1016/j.ijmecsci.2015.03.002
0020-7403/

[21], double-walled carbon nanotubes (DWCNTs) [18] and MWCNTs [42,43]. Besides, the tight-binding approach [44] and molecular dynamics simulation [45] were developed to investigate torsional buckling of SWCNTs. Other interesting phenomena of nano-structured materials that have been reported include the effects of interface [46–48], couple stress theory [46], strain gradient in torsion [47], and effective elastic moduli [48].

In the research works mentioned above, the classical quantities are directly replaced by the corresponding nonlocal quantities in the equilibrium equation to derive the nonlocal governing differential equation of motion. This model has been extensively used to study the mechanical properties of nanomaterials or nanostructures as described above. The differential nonlocal constitutive relation was transformed from the original integral nonlocal constitutive relation [3–6]. This integral-to-differential transformation imposes some restrictions which limit the applicability of the differential constitutive relation. One limitation is that the nonlocal boundary effects represented by the integration of a kernel function in the integral formulation are not perfectly matched or transformed in the differential form at a boundary. For this purpose, a new nonlocal finite element method (NL-FEM) with integral form of nonlocal elasticity [49–52] is developed in this paper and it is solved numerically for different cases in the context of statics and dynamics for torsional behavior of circular nanostructures.

Although there exist various scopes of research for transverse bending, vibration, and wave propagation of nonlocal nanostructures, very limited studies on torsional behaviors are available at present. Because torsional deformation and vibration in NEMS and some other nano-devices are frequently present, their effects and behavior should not be discounted. In this paper, angular deformation and free vibration of circular nanostructures in the presence of combined distributed torque and fixed external end torque based on nonlocal elasticity theory [49–52] are investigated. In addition, the differential form of nonlocal elasticity theory [3–6] is also used to obtain the analytical solution for static problem. To compare the accuracy of the present NL-FEM, the results obtained are compared with those predicted by the analytical solutions and other models numerically and qualitatively including the couple stress theory [46], the strain gradient model [47] and the Mori-Tanaka effective field model [48]. The new NL-FEM solutions are consistent and the conclusions are insightful with respect to the solutions of transverse bending and vibration of nanobeams and CNTs.

2. Nonlocal modeling and formulation

2.1. Kinematics

The static torsion of a fixed-free nanorod/nanotube with length L and subjected to a combined distributed torque $T(x)$ and fixed end torque T_0 is shown in Fig. 1.

For a circular nanostructure, the relation between the classical shear stress $\sigma'_{r\theta}$ and the shear strain γ at a point r_0 from the center can be expressed as

$$\sigma'_{r\theta} = G\gamma = Gr_0\frac{d\theta}{dx} \tag{1}$$

where G is the shear modulus and $\gamma = r_0(d\theta/dx)$ is the shear strain. The variation of torsional strain energy of the system is given by

$$\delta U = \int_V \sigma_{r\theta}\,\delta\gamma\; dV = \int_0^L \int_A \left(\sigma_{r\theta} r_0 \delta\left(\frac{d\theta}{dx}\right)\right) dA\; dx$$
$$= [T_{r\theta}\delta\theta]_0^L - \int_0^L \left(\frac{\partial T_{r\theta}}{\partial x}\delta\theta\right) dx \tag{2}$$

with

$$T_{r\theta} = \int_A r_0\sigma_{r\theta}dA \tag{3}$$

where $T_{r\theta}$ is called the stress resultant or torque. In the presence of twisting moment $T(x)$ and an end torque T_0, the variation of work done by this combined load is given by

$$\delta W = [T_0\;\delta\theta]_0^L + \int_0^L T(x)\delta\theta\; dx \tag{4}$$

Similarly, the variation of kinetic energy for torsional vibration is

$$\delta E_k = -\int_0^L \rho I_p\left(\frac{\partial^2\theta}{\partial t^2}\right)\delta\theta\; dx \tag{5}$$

where ρI_p is the mass polar moment of inertia.

2.2. Constitutive relations

2.2.1. Differential form

Unlike the classical theory, the nonlocal elasticity theory [3–6] assumes that the nonlocal stress tensor at a point $\mathbf{x}$ can be expressed as a result of the weighted average of the contribution of the stress field within the body in the following expression

$$\sigma(\mathbf{x}) = \int_V \alpha(|\mathbf{x}'-\mathbf{x}|,\tau)\sigma'(\mathbf{x}')dV(\mathbf{x}') \tag{6}$$

where $\sigma(\mathbf{x})$ and $\sigma'(\mathbf{x}')$ are the nonlocal and local stress, respectively. The weighting function is specified by a nonlocal modulus $\alpha(|\mathbf{x}'-\mathbf{x}|,\tau)$ which depends on a dimensionless length nanoscale

$$\tau = \frac{\sqrt{\mu}}{L} = \frac{e_0a}{L} \tag{7}$$

where $\mu = (e_0a)^2$ is the nonlocal parameter, e_0 is a material constant and a is an internal characteristic length such as lattice parameter, granular distance while L is an external characteristic length. In this paper, characteristic length L is taken as the length of the nanostructure. Although there has been no rigorous research on the estimate of this nonlocal parameter, it is suggested that this parameter can be determined by conducting a comparison of dispersion relation between the nonlocal continuum mechanics and molecular dynamic simulation or experimentally in an empirical sense through vibration frequency or buckling load measurement.

For a one-dimensional elastic thin structure, a differential form of the nonlocal constitutive relation of Eq. (6) can be expressed as [4]

$$\sigma_{r\theta} - (e_0a)^2\frac{d^2\sigma_{r\theta}}{dx^2} = \sigma'_{r\theta} \tag{8}$$

where $\sigma_{r\theta}$ and $\sigma'_{r\theta}$ are the nonlocal and classical shear stresses of structure respectively; and x is the axial coordinate.

2.2.2. Integral form

According to Eringen and his associates [49,50], the nonlocal theory differs from the classical relation only for the construction of stress-strain constitutive relation. Based on this assumption, they developed a simplified elasticity theory for linear homogeneous isotropic continuum and the stress-strain relation for such a continuum is indeed assumed in the form

$$\sigma(\mathbf{x}) = \mathbf{D} : \widehat{\varepsilon}(\mathbf{x}) \quad \forall\; x \in V \tag{9}$$

where V is the domain; $\mathbf{x}$ is a vector in this domain; $\mathbf{D}$ is the elastic moduli fourth-order tensor of isotropic local elasticity; $\sigma(\mathbf{x})$ is the second order tensor representing the stress field at $\mathbf{x}$; and $\widehat{\varepsilon}(\mathbf{x})$ is the second-order strain tensor representing the nonlocal strain field at $\mathbf{x}$ which is the sum of strain arising at $\mathbf{x}$ itself and the strain at $\mathbf{x}$ induced by strain arising at all $\mathbf{x}' \neq x$ in V. In general, the second contribution can be expressed in terms of integral which governs the nonlocal behavior of the material in the constitutive