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Mechanical response of a helical body to axial, torsional and radial strain

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ABSTRACT

The axial and torsional response of helical bodies as well as their inner coupling has been in depth analyzed, both from an analytical and a numerical modeling perspective. Herein, the radial deformation is appended as an additional kinematic degree of freedom. To this end, an existing analytical theory is employed and advanced so as to account for radial strain. The extended stiffness matrix entries are further verified with the use of a finite element model over a wide range of helix angular configurations. Overall, closed-form expressions of the 3×3 structural response of the helical body are provided, relating axial, torsional and radial strain to their force and moment resultants.

1. Introduction

Technology evolution and applications have been in a constant optimization process between engineering limits and human societies' needs. In the long history of load carrying structure's development, helices have appeared in various forms as springs, ropes, lifting cables or overhead power line cables. More recently, helical structures have found numerous applications in medicine, indicative examples being stents or scaffolds for tissue repair.

Even though the materials and structural configurations may vary depending on the chronological period and application purpose, their mechanical response can be generically characterized, providing thus the necessary basis for the analysis and design of any potential application. In the subsections to follow, a bibliographic review is provided, complemented by the content and purpose of the current work.

1.1. Helical structure models

Up to now, a large number of modeling schemes on the structural response of helical structures have been proposed, initially with analytical theories which appear in the literature at the first half of the twentieth century [1]. Analytical derivations have been complemented by several finite element models, enriching the helix structural behavior comprehension, as well as extending available hand-tools of the engineering community.

Analytical modeling schemes addressed the coupled axial and torsional response of helical structures as a gradual incorporation of different mechanical contributions. In particular, in early modeling developments, the axial stiffness of the helix cross section has been the one and only mechanism taken into account for the description of the helix response [2,3]. Later on, Machida and Durelli [4] incorporated the effect of the torsional stiffness of the helix cross section, their modeling approach was further elaborated by McConnell et al. [5]. Emphasis on the influence of the bending stiffness of the helix cross section was put by Costello [6] making use of linearized thin beam theory. Utting and Jones [7,8] accordingly presented a beam theory based model which they complemented by extensive experimental work on engineering strands. Raoof and Kraincanic [9] addressed the validity of existing approaches through parametric studies and experimental data comparisons. More specifically, limitations and advantages of thin rod models and orthotropic sheet models were highlighted in relation to the configuration and size of the helical strand to be analyzed. Finally, a thin beam theory model – the first to furnish a symmetric axial–torsional strain stiffness matrix – was presented by Sathikh et al. [10], its derivations grounded on four generalized strain parameters after the developments of Ramsey [11].

In the realm of computational modeling, several beam and volume element based approaches have appeared in the literature. To mention but a few, Jiang et al. [12–14] modeled helical geometry effects making use of a helical slice discretized with volume elements. Nawrocki and Labrosse [15] employed

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displacement based beam elements upon a Cartesian iso-parametric formulation, through which, the effect of different kinematic considerations on the structural response of simple engineering strands was studied. A homogenization approach for periodic beam-like structures was followed by Cartraud and Messager [16] with their modeling scheme likewise applied to simple engineering strands. The latter have been further analyzed with the use of three dimensional finite elements by Ghoreishi et al. [17], where limitations of analytical theories have been pointed out. In more recent works, Usabiaga and Pagalday [18] simulated helical and double helical configurations response when subject to tensile and torsional loads, with the use of beam elements. Finally, Imrak and Erdönmez [19] presented a three dimensional finite element analysis technique for the modeling of wire ropes with independent wire rope core (IWRC) both for simple and double helical geometries.

1.2. Structure of the present work

While the coupled axial and torsional response of helical bodies has been extensively analyzed, their radial deformation mode has been commonly disregarded. The present work provides an insight into the radial response of thin helical structures over the entire span of helical angles, providing closed-form solutions for the respective stiffness terms. To that extent, an existing, thin beam theory based analytical theory is advanced, so as to account for radial strain in Section 2. Thereafter, a finite element model is employed to further validate the extended stiffness matrix entries which are subsequently graphically depicted in Section 3. Finally, a discussion on the radial stiffness physical significance and relevance to practical applications is made (Section 4), followed by concluding and recapitulative remarks (Section 5).

2. Structural model

2.1. Helix geometry

The helix is described as a thin helical fiber, with its position vector being defined in a *Cartesian* basis as follows:

$$\mathbf{R}=\begin{Bmatrix} a\cos\varphi \\ a\sin\varphi \\ b\varphi \end{Bmatrix}, \quad \varphi=\frac{\ell}{\gamma}, \quad \gamma=\sqrt{a^2+b^2} \tag{1}$$

where a stands for the centerline position of the helix, while b and γ represent the rise along its central *Cartesian* axis and its *Curvilinear* length respectively, per unit angular evolution φ. Fig. 1 depicts the basic geometric parameters of a helical body along with a schematic representation of the deformation modes that are taken into consideration.

It should be noted that ℓ represents the length of the helix, its projection on the *Cartesian* axis Z is named as h, while θ describes the helix angle:

$$h=\ell\sin\theta, \quad a\varphi=\ell\cos\theta, \quad \theta=\arctan\left(\frac{b}{a}\right) \tag{2}$$

The force and moment resultants developed on the helix cross section upon loading are denoted as F_t, M_t and F_b, M_b respectively, following the tangential and binormal local helix axis.

2.2. Structural response

The deformation patterns considered for the helical body are namely axial, radial and torsional strain. The stiffness matrix connecting the axial, radial force and torsional moment resultants

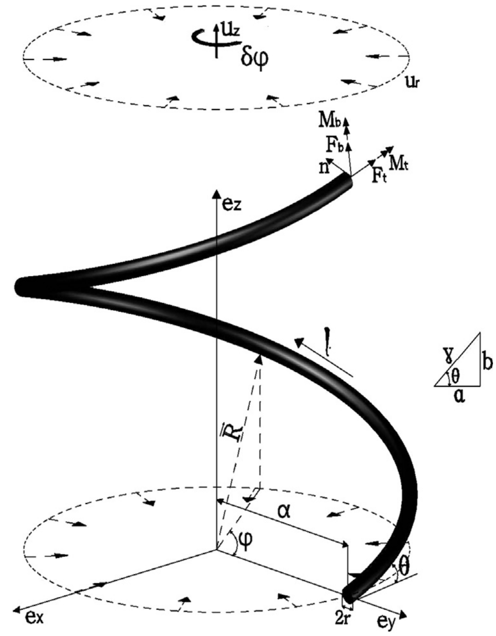

Fig. 1. Helix geometry.

with the preceding strains is below defined as:

$$\begin{bmatrix} F_z \\ M_z \\ F_r \end{bmatrix} = \begin{bmatrix} \kappa_{\epsilon_z\epsilon_z} & \kappa_{\epsilon_z\omega'} & \kappa_{\epsilon_z\epsilon_r} \\ \kappa_{\omega'\epsilon_z} & \kappa_{\omega'\omega'} & \kappa_{\omega'\epsilon_r} \\ \kappa_{\epsilon_r\epsilon_z} & \kappa_{\epsilon_r\omega'} & \kappa_{\epsilon_r\epsilon_r} \end{bmatrix} \begin{bmatrix} \epsilon_z \\ \omega' \\ \epsilon_r \end{bmatrix} \tag{3}$$

where the strain variables appearing at the right side of the equation are defined as follows:

$$\epsilon_r=\frac{\delta\alpha}{a}, \qquad \epsilon_z=\frac{\delta h}{h}, \quad \omega'=\frac{\delta\varphi}{h} \tag{4}$$

It should be pointed out that the typical description commonly employed in (3) is dependent on the helix geometry. Therefore, a normalized form of its entries is employed.

$$\begin{aligned} &\kappa^*_{\epsilon_z\epsilon_z}=\frac{\kappa_{\epsilon_z\epsilon_z}}{EA} \quad \kappa^*_{\epsilon_z\epsilon_r}=\frac{\kappa_{\epsilon_z\epsilon_r}}{EA} \qquad \kappa^*_{\epsilon_r\epsilon_r}=\frac{\kappa_{\epsilon_r\epsilon_r}}{EA} \\ &\kappa^*_{\epsilon_z\omega'}=\frac{\kappa_{\epsilon_z\omega'}}{EAa} \quad \kappa^*_{\epsilon_r\omega'}=\frac{\kappa_{\epsilon_r\omega'}}{EAa} \quad \kappa^*_{\omega'\omega'}=\frac{\kappa_{\omega'\omega'}}{EAa^2} \end{aligned} \tag{5}$$

where EA stands for the axial stiffness of the helical body cross section, with E being the Young's modulus and A the contained area ($A=\pi r^2$).

2.3. Radial force notion structural interpretation

The stiffness values relating axial strain ϵ_z and twist ω' with force F_z and moment M_z are independent of the number of helix windings. In contrast, an absolute radial force resisting a radial strain ϵ_r must increase proportional to the number of windings considered. To that extent, a normalized form of the radial force is introduced. Starting from equilibrium considerations on a thin helix, we note that a radial strain is balanced by a circumferential force component F_c. Considering the analogy of a thin-walled cylinder as the right plot of Fig. 2 illustrates, it is the circumferential line load Fc which balances the radial pressure, given as:

$$2a\pi bp=2F_c\rightarrow\overline{\sigma}_r=p=\frac{F_c}{\pi ab} \tag{6}$$

where $2a$ is the diameter and πb the height of the one-half helix winding used for formulating the equilibrium while $\overline{\sigma}_r$ the mean resisting stress to the applied strain. Averaged stresses for the axial force F_z and moment M_z can be accordingly defined by division