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Stability of imperfect columns with nonlinear connections under eccentric axial loads including shear effects

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ABSTRACT

An analytical method that evaluates the lateral stability and induced bending moments, shears, and second-order deflections in columns with initial geometric imperfections (i.e., initial curvature, out-of-plumb, and axial load eccentricities) and nonlinear semirigid connections subjected to axial load at both ends and lateral load at the top end is presented including the effects of shear forces and corresponding strains using three different approaches. The first two approaches are those by Haringx and Engesser that include the shear component of the applied axial force proportional to the angle of rotation of the cross section and to the total slope the along the member, respectively. The third approach is a simplified formulation based on the classical Euler theory that includes the effects of shear deformations but neglects the shear component of the applied axial force along the member. The proposed equations can be applied to columns with sidesway uninhibited, partially-inhibited and totally-inhibited. The proposed model is limited to prismatic columns subjected to eccentric axial loads at both ends and a lateral load at the top end causing bending about one of the principal axes with inelastic behavior taking place at the end rotational connections only. Five classical column cases and sensitivity studies are presented and discussed in detail. Analytical results indicate that the stability and second-order response of a column are highly affected by its initial imperfections, low shear stiffness relative to the bending stiffness, the stiffness of the lateral bracing, and by any changes in the stiffness and strength of the end connections.

1. Introduction

In structures made of beams and columns the lateral deflections along each element with respect to its cord (so called bowing or P–δ effects), the relative drift between the element ends (P–Δ effects), any initial geometric imperfections (i.e., initial curvature, out-of-plumb and eccentric axial loads), possible softening of its end connections (i.e., lost of their stiffness and strength) and shear force effects along the element all are coupled together causing nonlinear behavior with additional rotations, displacements and bending moments.

Razzaq and Calash [1] carried out an analytical study on the effects of biaxial semirigid connections on the inelastic response of hollow rectangular steel non-sway columns with biaxial crookedness and residual stresses. Papadrakakis and Loukakis [2] studied analytically the inelastic behavior of a prismatic column subjected to cyclic loading including the effects of an initial out-of-straightness, eccentricity of the applied axial load, and end restraints taking into consideration the plastic flow in the vicinity of hinging regions. Yau and Chan [3] developed a method to trace the equilibrium path of steel frames, allowing for geometrical, material and nonlinear connections up to the ultimate load using a beam-column element with springs connected in series. King and Chen [4] studied the nonlinear stability behavior of rigid and semirigid frames bent about the weak axis using a hardening plastic hinge method. Chan and Zhou [5] presented a method that includes the effects of initial imperfection on the column element stiffness matrix. Kim and Chen [6,7] presented procedures for 2D braced and unbraced steel frame analysis and design including gradual yielding associated with flexure, residual stresses, second-order effects and geometric imperfections. Chan et al. [8] developed a finite element procedure for large-deflection and inelastic analysis of imperfect steel frames with semirigid bases with various modes of initial imperfection. Surovek et al. [9] presented an approach that allows for the consideration of nonlinear connection response in framed structures with semirigid connections using commonly available elastic analysis software.

Eröz et al. [10] studied the effects of partial base fixity using the direct analysis method (DM) promoted by the 2005 AISC specifications for the stability design of steel frames. Xu and Wang [11] carried out parametric studies on the effects of initial imperfection and out-of-plumb on the lateral stability of unbraced plane frames. Rasmussen and Trahair [12] studied the elastic flexural buckling of doubly symmetric columns with oblique restraints under concentric axial loads causing coupling between the principal axis deflections

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Nomenclature

A *and* A_s gross area and effective shear area of column cross-section;
a initial camber at column midspan;
a_n peak value of the initial camber corresponding to n-wave;
E *and* G elastic and shear moduli;
e_a and e_b axial load eccentricities at ends A and B, respectively;
F applied horizontal force at the top B;
F_1 and F_2 traces on the M_aM_b plane defined by Eqs. (4a) and (4b) or (5a) and (5b);
h column height;
I moment of inertia of column cross-section in the plane of bending;
M_u ultimate moment capacity of end connection;
$M_a^* = M_a + Pe_a$ and $M_b^* = M_b - Pe_b$ induced end moments in the nonlinear end connections at A and B, respectively [+ clockwise when applied by the end connection to the column];
P applied axial load at both ends of column;
P_e $\pi^2 EI/h^2$;
n shape parameter of the end connection;
S_Δ stiffness of the lateral elastic bracing at the top end B;
u additional second-order lateral deflection along the column height caused by the applied axial load P;
u_0 initial column imperfection defined as $(x\Delta_o/h)$;
u_1 initial column imperfection (curvature) defined by a parabola symmetric about column midspan or by a series of sinusoidal waves;
$u_T = u + u_0 + u_1$ total lateral deflection;
Δ sway of column's top end with respect to a vertical line at end A;
Δ_o initial out-of-plumb of the column;
$\varphi^2 = Ph^2/\beta EI$ with $\beta = 1/(1 + P/GA_s)$ when using the Haringx approach;
$\varphi^2 = Ph^2/\beta EI$ with $\beta = 1 - P/GA_s$ when using the Engesser approach;
$\varphi^2 = Ph^2/EI$ with $\beta = 1/(1 + P/GA_s)$ when using the simplified Euler approach;
κ_{ai} and κ_{bi} an initial stiffness of the lumped bending nonlinear connections at ends A and B, respectively;
ψ_a^* and ψ_b^* rotations of the nonlinear end connections caused by the induced moments M_a^* and M_b^*, respectively.
ρ_i fixity factor $= 1/[1 + 3(EI/h)/\kappa_i]$

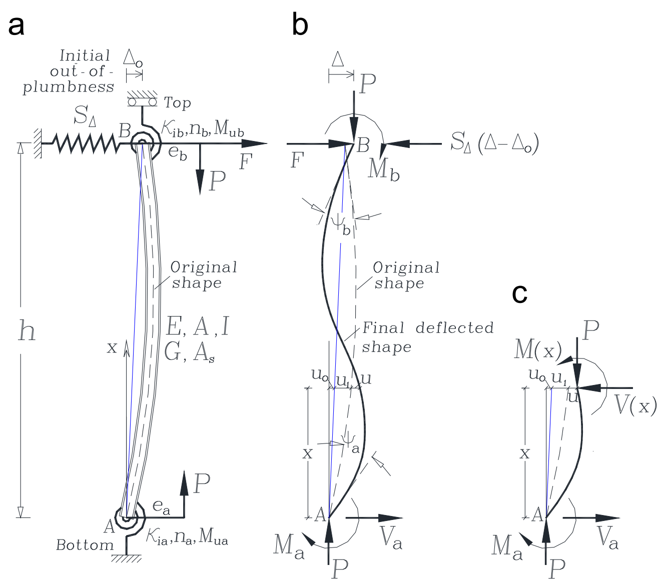

Fig. 1. Model of imperfect column with sidesway partially inhibited and nonlinear rotational end restraints: (a) structural model with eccentric axial loads applied at the column extremes; b) end moments, forces, rotations and deflections; and c) column segment including bending moments, shear and axial forces.

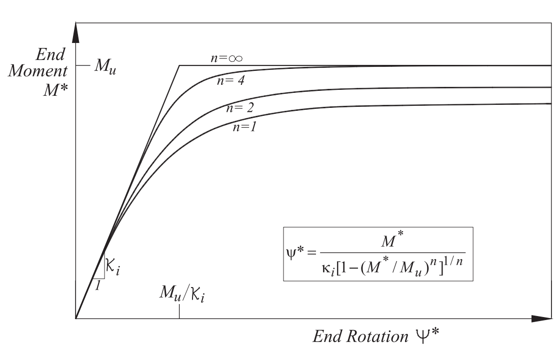

Fig. 2. Three-parameter power model used for the end rotational connections.

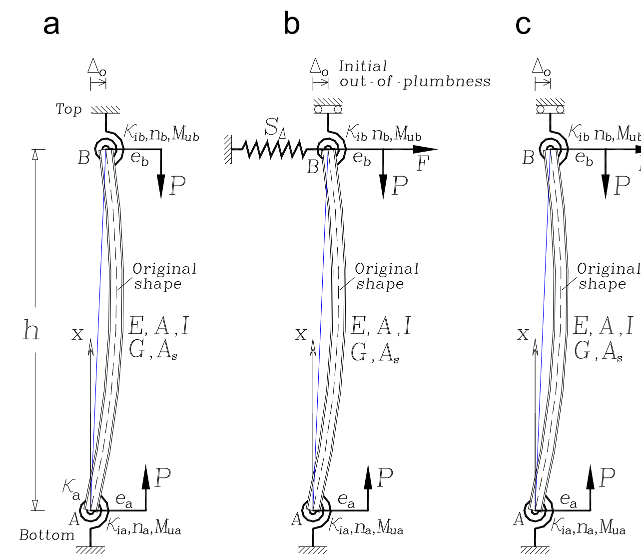

Fig. 3. Models of an imperfect column with nonlinear rotational end restraints and with sidesway: (a) totally inhibited (b) partially inhibited; and (c) uninhibited.

and rotations, and the flexural buckling mode involves simultaneous bending about both principal axes. More recently, Shayan et al. [13] studied the effects of geometric imperfections on the strength of steel frames using advanced geometric and material nonlinear analysis.

Smith-Pardo and Aristizabal-Ochoa [14–22] studied the effects of initial imperfections on the stability and second-order analysis of columns and frames with semirigid connections and developed closed-form expressions for the induced moments, shears, and lateral deflections in prismatic Euler–Bernoulli columns caused by any initial camber and out-of-plumb subjected to eccentric axial loads at both ends.

The main objective of this publication is to present an analytical method that evaluates the stability and the induced bending moments and second-order deflections of prismatic columns with initial geometric imperfections (initial curvature, out-of-plumb and axial load eccentricities) and non-linear semi-rigid connections. The