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Numerical and experimental analysis of elastic–plastic pure bending and springback of beams of asymmetric cross-sections

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ABSTRACT

We present a procedure for numerical computation of elastic–plastic bending and springback of beams with asymmetric cross-sections. Elastic-nonlinear hardening behavior of the material is assumed and both isotropic and kinematic hardening models are considered. The strains are described as a function of rotation and shift of the neutral axis and the curvature of the beam. Exact geometric expressions for large deflections and large rotations are taken into account during bending process. A complete loading history is taken into account including the effect of the local loading during the monotonic decrease of the load. Numerical examples confirm a strong influence of the load on the final and springback rotation of the neutral axis, its shift, and curvature of the beam for different cross-sections and materials. A custom made forming tool was designed and manufactured in-house to experimentally evaluate the proposed solution procedure. It is shown that relative difference between experimentally and theoretically predicted results of the final radius of curvature of the formed beam is $0.177 \pm 0.683\%$, if also the effect of pre-strain on elastic modulus is taken into consideration.

1. Introduction

Either as vital parts of load-bearing structures in mechanical and civil engineering or merely as an aesthetic feature in architecture, curved beams are most commonly made via some sort of forming process. V-bending, roll-bending, air and edge-bending, hydroforming, etc. are some of the examples of technological/manufacturing processes for obtaining the desired shape. The prediction of the (final) shape can be a complex task, especially because real-life materials often exhibit nonlinear mechanical response to loading.

In the forming process, the material undergoes elastic–plastic deformations. The plastic part of deformation changes the original shape of the object permanently, whereas the elastic part returns the deformed shape back towards initial configuration. Since a certain amount of elastic deformation is practically always present, the final shape of the object is not the same as the shape of the forming tool itself. A common way to deal with this problem is to add special techniques to reduce the effect of elastic recovery (also known as springback), such as extra features in radii, using smaller radii, or varying blankholder force in the forming process. These techniques reduce the effect of springback, but the formed part will always tend to springback by a certain amount.

In the available literature one can find a considerable number of papers devoted to this subject. Kosel et al. [1] presented an analytical solution of the simplified model for predicting the springback of beams made from material with an elastic-linear hardening response. The beams were subjected to repeated pure bending and unbending process and complete strain history was considered. The influence of axial force on the bending and springback of the elastic–ideal plastic beam was investigated by Yu and Johnson [2]. Johnson and Yu [3] developed formulas for springback of beams and plates undergoing linear work hardening. Springback of equal leg L-beams subjected to elastic–plastic pure bending was described by Xu et al. [4]. A theoretical model to predict the final geometrical configurations of wires made of different materials after loading and unloading was proposed by Baragetti [5]. Although analytical solutions can be obtained only for relatively simple problems. Their advantage is that they enable better insight and understanding of the problem and the influence of the process parameters. For more complex problems, however, the general practice is to refer to numerical techniques. Thus Li et al. [6] analyzed draw-bend tests of sheet metals using finite element modeling, where some of the results have been compared with experiments. The error associated with numerical through-thickness integration was investigated by Wagoner and Li [7]. The prediction model for springback in a wipe-bending process was developed by Kazan et al. [8] using artificial neural network approach together with the finite element method. Panthi et al. [9] analyzed and examined the effect of load on springback of a

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typical sheet metal bending process using a large deformation algorithm. Furthermore, Ragai et al. [10] investigated the influence of sheet anisotropy on the springback of drawn-bend specimens by means of experiments and finite element analysis. Vladimirov et al. [11] developed a finite strain model by combining both nonlinear isotropic and kinematic hardening, where for the integration of the equations a new algorithm based on an exponential map was used. An interesting phenomenon, a decrease of elastic modulus, can be observed in experiments on (e.g. metal) materials during plastic deformation. Ghaei [12] presented a numerical procedure which took into account also this effect. He implemented the elasto-plastic constitutive laws assuming elastic modulus as a function of effective plastic strain. The evolution of the elastic modulus with plastic deformation was also studied in other papers, see e.g. [13–20]. The change of elastic modulus during unloading is usually described by introducing the linear chord modulus [14,19,20] in many practical applications. However, experimental studies have shown that the elastic deformation during unloading is not perfectly linear. In work done by Sun and Wagoner [18] a new concept of quasi-plastic-elastic (QPE) strain was introduced within the continuum framework to model the nonlinear unloading behavior. It was shown that QPE concept is superior to the linear chord modulus in accurate prediction of springback in cases when the unloading stops at non-zero stresses.

These studies of elastic–plastic deformations of beams are mostly limited to symmetric cross-sections. Here, we present the solution procedure for elastic–plastic bending and springback of beams with asymmetric cross-sections. Elastic-nonlinear hardening behavior of the material is assumed and both isotropic and kinematic hardening models are considered. The strains are described as a function of rotation and shift of the neutral axis and the curvature of the beam. Exact geometric expressions for large deflections and large rotations are used during bending process. Strains, on the other hand, are considered to remain small. A complete loading history, including the effect of the local loading during the monotonic decrease of the load, is taken into account. Numerical examples confirm a strong influence of the load on the final and springback rotation of the neutral axis, its shift, and curvature of the beam for different cross-sections and materials. Generally, forming of beams with asymmetric cross-sections involves also torsional deformations, which we do not consider in our computations. Instead we find a special combination of forming parameters to constrain (to remove the effect of torsion from) an asymmetric rectangular L-beam to deform in one plane. Note that the presented solution procedure can easily be used for more complex shapes of cross-sections. We also present a custom-made forming tool, designed and manufactured in-house to experimentally evaluate the proposed solution procedure. Practically perfect planar shapes of the beams are obtained after forming, showing an excellent agreement with theoretical predictions, especially when the effect of the pre-strain on elastic modulus is considered.

2. Formulation of the problem

We consider a beam of asymmetric cross-section subjected to the bending moment $M(t)$ in direction α, as shown in Fig. 1, and assume an isotropic, homogeneous material which exhibits elastic-nonlinear hardening behavior. The yield point is defined by non-negative parameters σ^0 and ε^0 (cf. Fig. 3). Suppose that the beam is stress-free before loading and that the mechanical response can be described by

$$\sigma(\varepsilon;\varepsilon^0,\sigma^0)=\begin{cases} f_{\mathrm{e}}(\varepsilon) & \text{for } |\varepsilon|\le\varepsilon^0 \\ f_{\mathrm{p}}(\varepsilon;\varepsilon^0,\sigma^0) & \text{for } |\varepsilon|>\varepsilon^0, \end{cases} \tag{1}$$

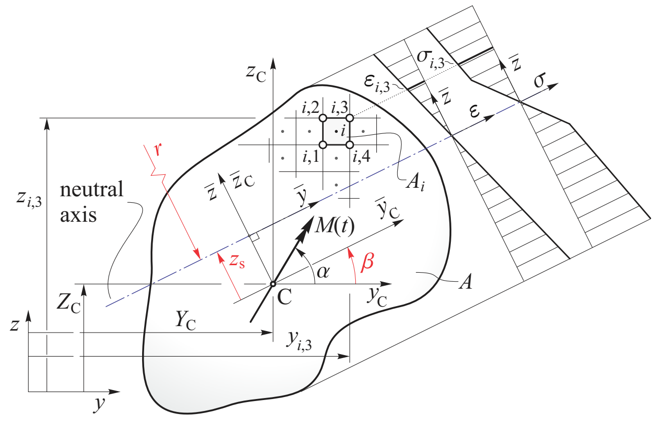

Fig. 1. Bending stress and strain state in the cross-section of the beam.

where $f_{\mathrm{e}}(\varepsilon)$ and $f_{\mathrm{p}}(\varepsilon;\varepsilon^0,\sigma^0)$ represent stress–strain response in the elastic and plastic regions, respectively. Both isotropic and kinematic hardening models are considered.

Exact geometric expressions for large deflections and large rotations are taken into account during bending process. Following the Euler–Bernoulli theory, valid for slender beams, a strain distribution over the cross-sectional area due to bending can be described by the following expression:

$$\begin{aligned}\varepsilon &= -\kappa\bar{z} \\ &= -\kappa(\bar{z}_{\mathrm{C}}-z_{\mathrm{s}}) \\ &= -\kappa(-(y-Y_{\mathrm{C}})\sin\beta+(z-Z_{\mathrm{C}})\cos\beta-z_{\mathrm{s}}),\end{aligned} \tag{2}$$

where $\kappa=1/r$ is the curvature of the beam (and r is its radius of curvature), β is the rotation of the neutral axis, z_{s} is the shift of the neutral axis from the centroid and $Y_{\mathrm{C}}, Z_{\mathrm{C}}$ are the coordinates of the centerline taken from the reference coordinate system y–z (see Fig. 1). The neutral axis is found from the no-strain condition, $\varepsilon\equiv 0$. In the case of a linear elastic beam the neutral axis goes through the centroid of the cross-section.

Equations of static equilibrium of the beam $\sum_i\vec{F}_i=0$ and $\sum_i\vec{M}_i=0$ and static equilibrium of the infinitesimal element yield $dM_y/ds=0$ and $dM_z/ds=0$ (M_y, M_z are constants) in the case of pure bending, where s is a curvilinear coordinate along the length of the deformed beam (note that curvature $\kappa=d\vartheta(s)/ds$, where $\vartheta(s)$ is the angle of inclination of the plane, tangent to the beam's neutral surface, at the local coordinate s). The stress resultants are

$$N=\int_A\sigma\,dA,\quad M_y=-\int_A\sigma z\,dA,\quad M_z=\int_A\sigma y\,dA, \tag{3}$$

where N represents inner axial force, M_y and M_z are inner bending moments in directions of y-axis and z-axis, respectively, and σ is the normal stress. Due to the nonlinearity of the problem, the computation of integrals in Eq. (3) is numerical. The integration domain (the cross-section) is divided into n rectangular elements. A generic element A_i, $i\in\{1,2,\dots,n\}$ (cf. Fig. 1) is defined by four nodes in which the mechanical properties are known. Since a complete loading history has to be considered and the effect of the local loading during the monotonic decrease of the bending moment occurs (and vice versa in monotonic increase of the load in non-virgin material), the bending moment $M(t): 0\to M_{\max}$ and $M_{\max}\to 0$ is applied incrementally, as shown in Fig. 2. Here variable t represents a pseudo-time, which is used to follow successive increments of the load (and stress and strain).

The mechanical state corresponding to the current load $M(t_0)$ thus includes a complete loading history. Since the strain in each node of the cross-section $\varepsilon_j^{t_0-\Delta t}$, $j\in\{1,2,\dots,n_{\mathrm{N}}\}$ (where n_{N} is the