International Journal of Mechanical Sciences 90 (2015) 102–112

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Dynamic analysis of two-layer composite beams with partial interaction using a higher order beam theory

Guanghui He*, Xiao Yang

Department of Civil Engineering, Shanghai University, Shanghai 200072, China

A R T I C L E I N F O

Article history:
Received 26 July 2014
Received in revised form
16 September 2014
Accepted 23 October 2014
Available online 31 October 2014

Keywords:
Higher order beam theory
Composite beams
Seismic response
Moving load
Finite element method

A B S T R A C T

Kant's higher order beam kinematics, which takes into account both the longitudinal and transverse higher order deformation of beam, is applied to the dynamic model of two-layer partial interaction composite beams. To establish and solve the mathematical problem, the finite element for the composites is then formulated using the principle of virtual work. Furthermore, the finite element of Timoshenko composite beam model, for the purpose of comparison, is also given in this paper. The numerical performance and reliability of the proposed finite elements are verified through the comparison with the results of ABAQUS using the plane stress model and those based on Reddy's higher beam theory and classical beam theories from the literature. Besides, the responses to the seismic and moving load of the proposed composite beam model are also presented, and subsequently the influences of parameters including damping ratio, velocity of moving load, slenderness ratio and interfacial stiffness on the mechanical behavior are studied. Numerical results show that the present higher order composite beam model can achieve higher accuracy on the dynamic analyses than the classical and Reddy's models, and the impact effects of moving load together with the partial interaction between sub-layers should be considered.

1. Introduction

Many advantages can be found in the composite structures over their isotropic counterparts, e.g. the steel–concrete composite structures possess a higher ratio of strength to weight than the conventional reinforced concrete structures. As a result, this type of structure has received a wide range of applications in civil engineering, aerospace, automotive, etc. Usually, the flexible shear studs are used in the interface to connect each sub-layer of composite beams, which may cause the interfacial slip between sub-layers. To study the slip effects, the first two-layer steel–concrete composite beam model, taking into account the partial interaction, was proposed by Newmark et al. [1] in 1951. In the model, each sub-layer was described by small deformation Euer–Bernoulli beam theory (EBT), which neglected the shear deformation throughout the beam body. This assumption, however, implies that the mechanical behavior may not be precisely predicted for deep composite beam structures [2] due to the significant shear effect; thus the necessity arises to refine their model.

Fairly recently, the Newark's model has been refined by a great deal of investigators [3–8] to take into account the shear deformations using the Timoshenko beam kinematics. Either by finite element method (FEM) or analytical one, the linear static analyses of composite beams were performed by Refs. [6–9]. For the dynamic problems, many studies can also be found to research the linear dynamic characteristcis [5,10,11] and nonlinear free vibration problem [12]. However, what has to be noted is that the shear correction factor introduced by Timoshenko beam theory (TBT) is attributed to the cross-sectional geometry of each sub-layer as well as the shearing stress around the section [13], i.e. this factor is no longer constant during the deformation for the sub-layers of composite beams, and it was also demonstrated by He and Yang [2]. To avoid the problem caused by the correction factor, higher order beam theory (HBT) has received much attention [13–15], whose kinematics is even more elaborate than that of the TBT. Using Reddy's [16] HBT, where a third order polynomial is taken to approximate the axial displacement of sub-layers, Chakrabarti et al. [14,15] studied the static response of two-layer composite beams within linear and elastic range, and Chakrabarti et al. [13] extended it to the range of dynamics problems by FEM. Besides, Subramanian [17] developed a displacement based finite element for free vibration analysis of composite laminated beams, and outlined the analytical procedure for free vibration of beams using two types of HBTs; Li et al. [18] developed an exact finite element to conduct the free vibration analysis of laminated beams using hyperbolic shear deformation theory, and Vo and Thai [19] performed the static analysis of

* Corresponding author.
E-mail address: flamehe@shu.edu.cn (G. He).

http://dx.doi.org/10.1016/j.ijmecsci.2014.10.020
0020-7403/

laminated beams by FEM based on various refined higher order deformation theories [20].

Most of the HBTs, including the Reddy's HBT, tended to neglect the transverse deformation of the beam, thereby, neglected the transverse normal strain and stress. To our point of view, capturing the transverse normal stress of sub-layers caused by the interfacial pressure or tension may reduce the gap from beam model to plane stress model. Trying to achieve this, Kant's [21–23] HBT is used in this study, where both the longitudinal and transverse higher order displacements are considered by approximating displacement in these two directions as third and second order polynomials, respectively. Subsequently, a dynamic model for two-layer composite beams, whose sub-layers follow Kant's HBT, is proposed by virtue of the principle of virtual work. And the finite element for the transient response and free vibration analyses is also developed. For the purpose of comparison, the finite element formulation for two-layer composite beam model based on TBT is also given. The improvement of the incorporated transverse deformation on the accuracy is examined through the comparisons among the results of composite beams using plane stress, Reddy's HBT and classical beam theories for the free vibration problem. In addition, the seismic analyses are carried out to investigate the performances of the present composite beam model on the transient response analysis. Finally, the responses of the composite beams to the moving load are also studied, especially to investigate the effects of the moving velocity and damping ratio of structure on the composite beam deflection.

2. Formulations

2.1. Description of problems and assumptions

Let us consider a straight, planar, two-layer composite beam with possibly different cross-sections and materials including flexible shear connectors uniformly smeared over the interface. Sub-layers with overall span L, as is shown in Fig. 1(a), are marked with c and s. The layers are placed in Cartesian coordinate systems x–z_c and x–z_s, which originate from the centroid of each layer at the left end. And depths h_1 and h_2 are set to denote the centroid-interface distances of layers c and s, respectively. Both the longitudinal and transverse displacement field assumption for each layer are presented in Fig. 1 (b), where functions U_i and W_i, i=c, s denote the axial and transverse displacements throughout layers c and s, respectively. From Fig. 1(b), it can be foreseen that the transverse normal stress is going to influence the mechanical behavior, and the higher order shearing stress over the depth can be captured.

The basic assumptions of the present composite beam model are (1) the materials of the beams are linear elastic, and the displacement and rotation of the beam are small; (2) the tangential and normal interface resistances are proportional to the relative slip and separation at the interface, and (3) the displacements of each layer follow the present kinematics, as is shown in Fig. 1(b).

2.2. Kinematics

According to the hypothesis of small deformation Kant's higher order displacement assumption [22] (see Fig. 1(b)), axial displacement fields for the upper layer c and lower layer s can be expressed in a form of

$$U_i(x, z_i) = \sum_{n=0}^{3} u_{i\,n}(x) z_i^n, \quad i = c,\ s \tag{1}$$

and the transverse displacement fields can be formulated as

$$W_i(x, z_i) = \sum_{n=0}^{2} w_{i\,n}(x) z_i^n, \quad i = c,\ s \tag{2}$$

where U_i and W_i are the axial and transverse displacements of layer i at arbitrary point of initial configurations, respectively; $u_{i\,n}(x)$ and $w_{i\,n}(x)$ are the basic unknowns to be solved.

Theoretically speaking, there can be more terms of higher order displacement components in Eqs. (1) and (2) so as to reduce the gap from the plane stress model to the higher order beam model, according to the theory of Taylor-series expansion. However, the computation burden is to increase with the increase of number of higher order terms in Eqs. (1) and (2). Therefore, determining the reasonable number of such terms is of significance. As is known that the distribution of shearing stress over the beam depth is very close to parabola in the plane stress model, the axial and transversal displacements of the beam are chosen, respectively, as third and second order polynomials, in order to ensure the parabolicly distributing shearing stress.

The basic unknowns of the above kinematics are independent, which is different from Reddy's HBT, where his basic unknown terms are partly [13] or completely [16] eliminated by satisfying the shearing stress boundary conditions on the cross-section edge. Moreover, the transverse displacement is assumed to be constant at the same cross-section in Reddy's HBT [16], thereby, the transverse normal stress is neglected, while the present model, as is expressed in Eq. (2), assumes that the transverse displacement varies as a second order polynomial, which can capture a linearly distributing transverse normal stress.

2.3. Finite element formulation

According to the principle of virtual work, the transient dynamic problem of the present composite beams can be formulated as

$$\sum_{i=c,\,s} \int_{V_i} (\sigma_{i\,x}\delta\varepsilon_{i\,x} + \sigma_{i\,z}\delta\varepsilon_{i\,z} + \tau_i\delta\gamma_i)\mathrm{d}V_i + \int_0^L (k_u u_{cs}\delta u_{cs} + k_w\, w_{cs}\delta w_{cs})\mathrm{d}x$$
$$= \sum_{i=c,\,s}\left[\int_{V_i}\left(-\rho_i \ddot{U}_i \delta U_i\right)\mathrm{d}V_i + \int_{V_c}\left(-\rho_i \ddot{W}_i \delta W_i\right)\mathrm{d}V_i\right] + \int_0^L [q(x,\ t)\delta W_c]\mathrm{d}x \tag{3}$$

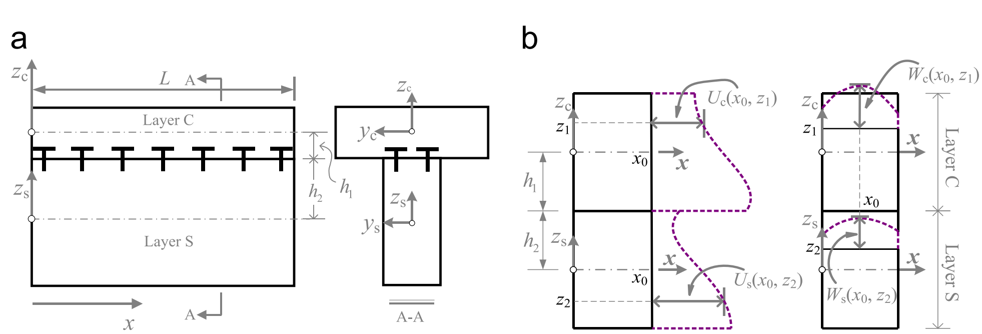

Fig. 1. Profile of partial-interaction composite beams and axial displacement field of higher order composite beams. (a) Elevation and cross-section. (b) Displacement assumption.