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Nonlinear dynamical behaviour of geometrically imperfect microplates based on modified couple stress theory

Hamed Farokhi [a,1], Mergen H. Ghayesh [b,*,1]

[a] Department of Mechanical Engineering, McGill University, Montreal, Quebec, Canada H3A 0C3
[b] School of Mechanical, Materials and Mechatronic Engineering, University of Wollongong, NSW 2522, Australia

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ABSTRACT

The nonlinear dynamical behaviour of a geometrically imperfect microplate is examined based on the modified couple stress theory. The microplate is modelled by means of the von Kármán plate theory and Kirchhoff's hypotheses retaining all in-plane and out-of-plane displacements and inertia. An initial imperfection in the out-of-plane direction is taken into account and the equations of motion for the in-plane and out-of-plane motions are obtained by means of an energy method based on the Lagrange equations. This operation gives three sets of second-order nonlinear ordinary differential equations with coupled terms for two in-plane motions and one out-of-plane motion. These sets are transformed into double-dimensional sets of first-order nonlinear ordinary differential equations which are solved numerically through use of a continuation technique. Apart from the nonlinear analysis, an eigenvalue analysis is also conducted to obtain the linear natural frequencies of the system with different amplitudes of the geometric imperfection. The effect of the amplitude of the geometric imperfection and thickness of the microplate as well as the forcing frequency on the response of the system is highlighted. Finally, a comparison is made between the responses of the system based on the modified couple stress and classical continuum mechanics theories so as to highlight the importance of taking into account small-size effects.

1. Introduction

Recently, there has been a growth in the applications of continuous machine components of microscale dimensions [1–7], especially microplates, for example, in biosensors, biomechanical organs, microactuators, microswitches, and vibration and shock sensors. Due to their widespread application, the analyses on the motion characteristics of this class of systems have received a considerable attention in recent years [8–14]. Experimental investigations [15–16] showed that microstructures display strange size-dependent deformation behaviour; classical continuum theories are not capable of predicting this size-dependent behaviour. Hence, new continuum theories such as the strain gradient and modified couple stress theories have been developed to predict the size-dependent deformation behaviour of microstructures theoretically. The classical couple stress theory, developed by Mindlin and Tiersten [20] and Toupin [21], consists of two higher-order stress constituents in addition to the classical stress components. About 40 years later, Yang et al. [22] proposed the modified couple stress theory and facilitated the application of the classical couple stress theory by considering only one material length-scale parameter together with two classical material constants; in comparison to the strain gradient theory, the modified couples stress theory requires only one length-scale parameter.

There are few investigations in the literature which studied the dynamical behaviour of microplates. Most of these investigations are carried out employing *linear* mathematical models. For example, Lazopoulos [23] analyzed the linear bending of thin microplates on the basis of the strain gradient elasticity by taking into account the surface energy. Wang et al. [24] developed a size-dependent microplate model by means of the Kirchhoff theory based on the strain gradient elasticity theory. Jomehzadeh et al. [25] employed the modified couple stress theory to derive the linear out-of-plane equation of motion of a microplate so as to study the linear oscillations of the system. Hashemi and Samaei [26] examined the linear buckling behaviour of microplates subjected to an in-plane excitation, based on the nonlocal Mindlin plate theory. Zhang et al. [27] analyzed the static bending, free oscillations and buckling behaviour of size-dependent Mindlin microplates based on the modified couple stress theory. Nabian et al. [28] examined the stability of a functionally graded microplate subjected to hydrostatic and electrostatic pressures. Roque et al. [29] contributed to the field by investigating the bending of a shear deformable microplate on the

* Corresponding author.
E-mail address: mergen@uow.edu.au (M.H. Ghayesh).
[1] Both authors contributed equally to this work.

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basis of the modified couple stress theory by means of a meshless method. Ramezani [3] employed the strain gradient elasticity theory in order to develop a first-order shear deformation microplate model. Ashoori-Movassagh and Mahmoodi [30] modelled the linear out-of-plane motions of a microplate on the basis of the modified strain gradient elasticity theory through use of the extended Kantorovich method (EKM). Li et al. [31] obtained the bending behaviour of a simply supported bi-layered square Kirchhoff microplate. Ke et al. [32] investigated the bending, buckling and oscillations of size-dependent functionally graded annular microplates. These studies were pursued and extended in few papers taking into account *nonlinearities*. For instance, Asghari [33] derived the size-dependent equations of motion of microplates on the basis of the modified couple stress theory. Thai and Choi [34] developed a size-dependent model of functionally graded Kirchhoff Mindlin plates employing the modified couple stress theory. Ansari et al. [35] continued the investigations by examining the nonlinear free oscillations of functionally graded Mindlin microplates on the basis of the modified couple stress theory. Ke et al. [36,37] examined the postbuckling and nonlinear free oscillations of functionally graded annular plates based on the modified couple stress theory. In both of the valuable studies in Refs. [33,34], only the equations of motion were derived and no solutions for the *nonlinear dynamical behaviour* were provided; moreover, in Refs. [35,37], the nonlinear *free* oscillations were examined in the absence of external forces. In addition, in all of the aforementioned valuable investigations, the microplate is assumed to be *perfectly flat*. Microplates with *initial geometric imperfections* (i.e., *initial curvature*), on the other hand, are highly possible to be produced due to an improper manufacturing process. Moreover, they are present in MEMS applications, such as in microshutters, microvalves, band-pass filters, and microswitches. The current paper is the first which examines the nonlinear size-dependent behaviour of geometrically imperfect microplates subject to an external excitation load.

To the authors' best knowledge, there is no study in the literature which studied the nonlinear size-dependent dynamical behaviour of geometrically imperfect microplates subject to an external excitation force; the current paper is the first to do so. The geometrically imperfect microplate is modelled based on the modified couple stress theory by means of the von Kármán plate theory as well as Kirchhoff's hypotheses, retaining all the *in-plane* and *out-of-plane* displacements and inertia; it is also the first time that all the in-plane and out-of-plane displacements and inertia are retained in the nonlinear analysis of microplates. The initial imperfection is modelled by an initial deflection in the out-of-plane direction. The size-dependent potential energy and the kinetic energy of the system are obtained as functions of the displacement field and inserted in the Lagrange equations resulting in three sets of second-order nonlinear ordinary differential equations with coupled terms for two in-plane and one out-of-plane motions. These equations are then transformed into six sets of first-order nonlinear ordinary differential equations by means of a change of variables and then solved through use of the pseudo-arclength continuation technique [38]. A high-dimensional discretized model is employed to capture almost all modal interactions and to ensure accuracy. The effect of different system parameters including the amplitude of the geometric imperfection is examined. The influence of the small-size parameter on the response of the system is also highlighted.

2. Equations of motion and methods of solution

The schematic representation of the system is shown in Fig. 1, which is a rectangular microplate with in-plane dimensions a and b in the x and y directions, respectively, and thickness h. Let (O; x,

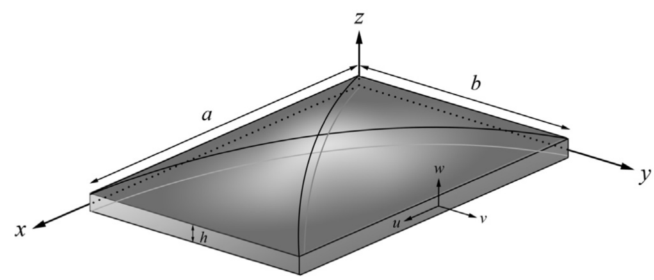

Fig. 1. Schematic representation of an initially imperfect rectangular microplate.

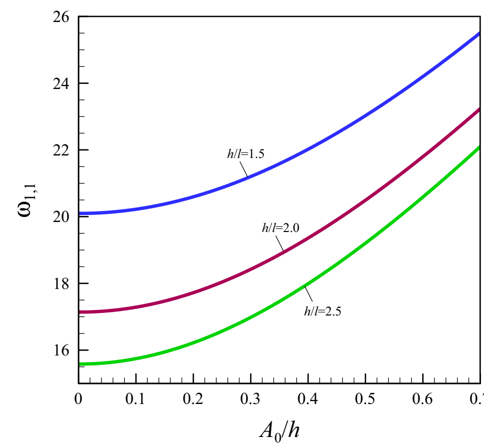

Fig. 2. The variation of the first dimensionless linear natural frequency of the out-of-plane motion with the amplitude of initial imperfection for different thickness ratios.

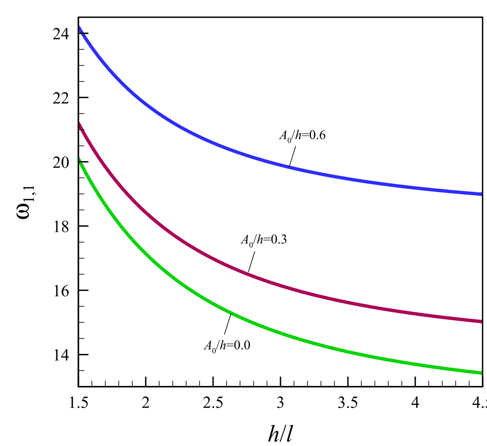

Fig. 3. The variation of the first dimensionless linear natural frequency of the out-of-plane motion with thickness for different amplitudes of initial imperfection.

y, z) be a rectangular Cartesian coordinate system with the origin O at one corner. The displacement field is denoted by $u=u(x,y,t)$, $v=v(x,y,t)$, and $w=w(x,y,t)$, showing the displacements of each point of the mid-plane of the microplate in the x, y, and z directions from the static equilibrium, respectively; t represents time. The microplate is subjected to a distributed harmonic force per unit area, $f_1 \cos(\omega t)$, in the z direction; it should be noted that f_1 is in N/m^2 and ω in rad/s. The initial curvature is in the positive out-of-plane direction and denoted by $w_0(x,y)=A_0 \sin(\pi x/a)\sin(\pi y/b)$.

The strain energy of the microplate, based on the modified couple stress theory developed by Yang et al. [22], can be