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Estimating constitutive equation of structural steel using indentation

Thai-Hoan Pham [a], Jung J. Kim [b], Seung-Eock Kim [c,*]

[a] Deptartment of Civil and Environmental Engineering, Sejong University, Seoul, South Korea
[b] School of Architecture, Kyungnam University, Changwon-si, South Korea
[c] Deptartment of Civil and Environmental Engineering, Sejong University, 98 Gunja-dong, Gwangjin-ku, Seoul 143-747, South Korea

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ABSTRACT

Unlike other metal materials, structural steels show plastic plateau in their constitutive equations. The plastic plateau continues from yield strain to the starting-point of strain hardening. In this study, the constitutive equations of structural steels, including the plastic plateau, were extracted from the indenting load to depth curves of indentation experiments. While conventional equations were used to determine the elastic modulus of steel, new equations were developed to determine the yield strength and the strain hardening exponent of steel from the indentation results by considering the plastic plateau. Two dimensionless relationships between the constitutive properties of steel and the characteristics of indenting load to depth curves were derived. A finite element (FE) model to simulate indentation was developed and FE analyses for 576 combinations of steel constitutive properties were conducted. The dimensionless relationships were then established in polynomial equations by regression analyses of the FE results. Indentation and uniaxial tensile loading tests were also performed to verify the FE model and the proposed equations to extract the constitutive properties using indentation.

1. Introduction

Indentation technique has been used to obtain mechanical properties of materials [1–15]. While the most popular method to extract the elastic modulus E from the indentation results is Oliver and Pharr's (O&P) method [1], many other methods have been provided to determine the plastic properties of materials [2–15]. Especially for the constitutive equations of elastic hardening materials such as alloyed engineering metals, as shown in Fig. 1, plastic characteristics in their constitutive equations are described with two plastic properties of the yield strength σ_y and the strain hardening exponent n [2–8] such as

$$\sigma = \begin{cases} E\varepsilon & (\varepsilon \le \varepsilon_y) \\ \sigma_y\left[1+E(\varepsilon-\varepsilon_y)/\sigma_y\right]^n & (\varepsilon \ge \varepsilon_y) \end{cases} \qquad (1)$$

One of the practical methods used to extract the plastic characteristics from the indentation results is the dimensional analysis [2–8]. The dimensional analysis has provided the basic relationship between the elastoplastic properties of the indented materials and the corresponding indenting load to penetration depth curves. While these studies focused on determining the constitutive equations consisting of elastic and power-law hardening parts in Eq. (1), the constitutive equation of structural steel illustrated in Fig. 2 includes plastic plateau continuing from yield strain ε_y to the starting-point of strain hardening ε_{st}. In order to reduce the complexity of severely disobeying power law elastic–plastic stress–strain behavior, the plastic plateau in constitutive equations of true stress–strain behavior (solid curve between ε_y and ε_{st} in Fig. 2) can be assumed to be a perfectly plastic plateau description (dotted line). The constitutive equation of structural steel can now be expressed as

$$\sigma = \begin{cases} E\varepsilon & (\varepsilon \le \varepsilon_y) \\ \sigma_y & (\varepsilon_y < \varepsilon < \varepsilon_{st}) \\ \sigma_y\left[1+E(\varepsilon-\varepsilon_{st})/(\alpha\sigma_y)\right]^n & (\varepsilon \ge \varepsilon_{st}) \end{cases} \qquad (2)$$

By comparing Eq. (2) with Eq. (1), one more plastic property α, where it is defined as ε_{st} divided by ε_y, is necessary to define the constitutive equation of structural steel. In Kato's study [16], the tensile loading test results from 36 specimens of SS41 and SM50A, which are the most popular structural steels, showed that the factor α varies in the range from 7 to 23.

Indentation is currently being used in many engineering fields including biomedical, civil, mechanical and material engineering [17–24]. Nanocharacterization of mechanical properties of materials using indentation and nanoscratch [18] has become important tools for providing quantitative evaluation of materials [19]. Extensive investigations using the indentation technique have been performed in the last decade on many materials [20–24].

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[*] Corresponding author. Tel.: +82 2 3408 3291; fax: +82 2 3408 3332.
E-mail address: sekim@sejong.ac.kr (S.-E. Kim).

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In this study, the constitutive equations of structural steel including plastic plateau were established from indentation results. While a conventional method was used to determine the elastic modulus E, new equations were developed to determine the yield strength σ_y and the strain hardening exponent n by considering the variation of α that could represent the starting-point of hardening and the length of plastic plateau. The dimensionless relationships between the constitutive properties of steel and the characteristics of indenting load to depth curves were derived. A finite element (FE) model to simulate indentation was developed and verified with the observations of indentation and uniaxial tensile loading experiments of structural steel specimens. FE analyses for 576 combinations of steel constitutive properties were conducted. The results of FE analyses were used for regression analyses to establish the dimensionless relationships in polynomial equations. Using the polynomial equations and the characteristics of indenting load to depth curves from indentation experiments, the constitutive properties were extracted for the α range from 7 to 23. The determined constitutive equations that were compared with them were observed from uniaxial tensile loading experiments for verification of the proposed polynomial equations. The limitations of the proposed equations were also discussed.

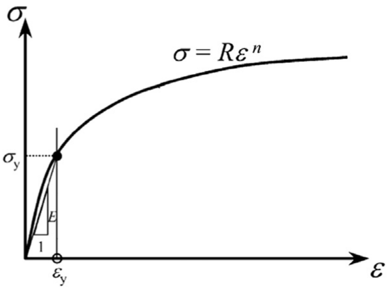

Fig. 1. Power law elastic–plastic stress–strain behavior of metals [2–8].

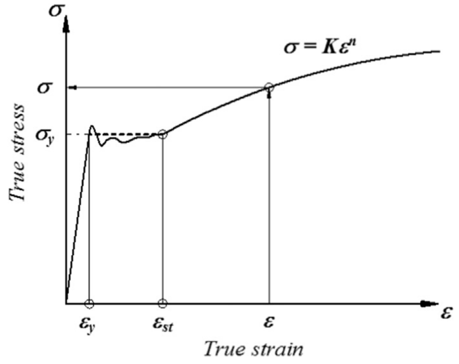

Fig. 2. Schematic illustrations of a typical constitutive description of structural steel (solid curve), and a perfectly plastic plateau description (dotted line).

2. Experiments

Indentation and uniaxial tensile loading tests were performed on two types of structural steel, SS400 and SM490 [25].

2.1. Specimen preparation

For indentation, 20 mm × 12 mm × 8 mm cubic specimens were prepared. The specimens were then mounted in epoxy matrix and polished with different silicon carbide papers, poly diamond particles and colloidal silica in order to obtain a flat and smooth surface. This procedure was achieved in seven stages of increasing fineness with the last one being in the range of 40 nm. For tensile loading tests, specimens were made according to ASTM standard [26]. Fig. 3 shows a SS400 steel specimen for tensile test and the indentation sample after being mounted and polished.

2.2. Tests

Indentation was performed at room temperature using a Nano Hardness Tester made by Swiss Center for Electronics and Microtechnology with a Berkovich indenter, which can be modeled as an

Table 1
Tensile loading test results of each material.

	SS400				SM490			
	E (GPa)	σ_y (MPa)	n	α	E (GPa)	σ_y (MPa)	n	α
S1	210.2	306.9	0.203	17.6	212.3	353.7	0.267	13.5
S2	213.2	305.6	0.205	18.0	212.1	354.3	0.262	13.4
S3	208.4	307.2	0.205	18.7	210.8	352.6	0.268	13.4
Ave.	**210.6**	**306.6**	**0.204**	**18.1**	**211.7**	**353.5**	**0.266**	**13.4**

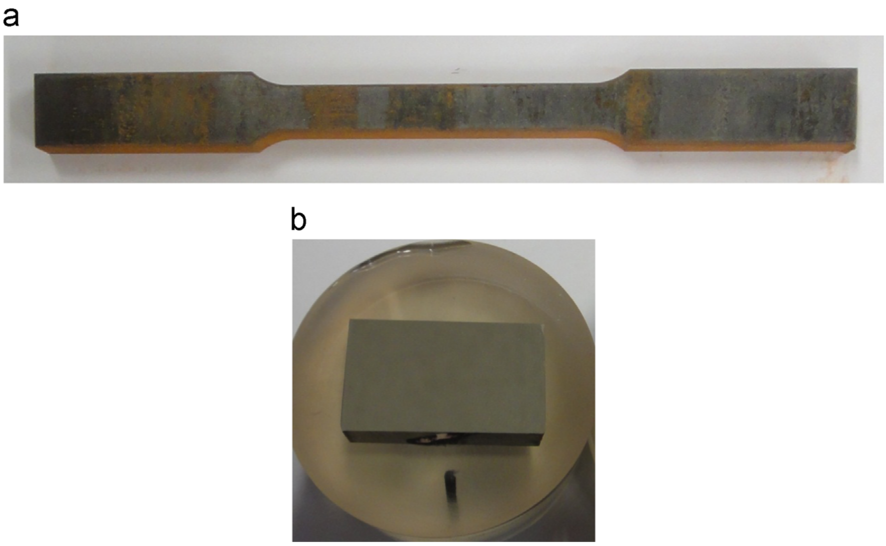

Fig. 3. (a) Specimen for tensile loading tests and (b) indentation sample.