International Journal of Mechanical Sciences 90 (2015) 246–250

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Bödewadt flow and heat transfer over a stretching stationary disk

Mustafa Turkyilmazoglu *

Department of Mathematics, Hacettepe University, 06532-Beytepe, Ankara, Turkey

ARTICLE INFO

Article history:
Received 26 May 2014
Received in revised form
12 October 2014
Accepted 27 October 2014
Available online 31 October 2014

Keywords:
Bödewadt flow
Radial stretching
Shear stress
Heat transfer

ABSTRACT

The three-dimensional motion of flow due to the rotation of a viscous fluid at a sufficiently large distance from a stationary disk, the so-called classical Bödewadt boundary layer flow, is extended in this paper for the first time in the literature to the case where the stationary disk is permitted to uniformly stretch in the radial direction. The effects of such a stretching mechanism on the flow and heat characteristics are of present concern for this physical problem. A conventional transformation is shown to lead to a set of similarity equations which are coupled and highly nonlinear. It is found that the traditional Bödewadt boundary layer is greatly altered under the influence of radial stretching of the wall. As a consequence of reduction in the momentum boundary layer, the thickness of the thermal boundary layer is also observed to considerably decrease even for moderate strength of stretching. This outcome is prominently important from the technological point of view since the radial stretching may serve to cool down the system in practical applications.

1. Introduction

A great deal of attention has been paid in the past decade to the traditional Von Karman flow and heat transfer, for which the motion is driven by a rotating disk. Its cousin problem constitutes one of the classical problems of fluid mechanics such that the motion is superimposed owing to the fluid rotating with a uniform angular velocity at a larger distance from a stationary disk. However, although this case also possesses theoretical as well as practical importance, for example in understanding the dynamics of tornadoes and hurricanes, and rotor–stator systems in turbines, less research has been conducted to understand its physical insight. Therefore, the current work is devoted to the three-dimensional revolving flow motion and heat induced by a stationary disk, but unlike to the classical case as considered in the past studies, a disk with a uniform stretching in the radial direction is taken into account here for the first time in the literature.

Bödewadt boundary layer flow occurs due to a rotating flow over a stationary disk and it represents a full analytical solution of the Navier–Stokes equations, since it was first theoretically investigated by Bödewadt [1]. For the example of the conventional rotating disk problem [2], the fluid forced outwards by the centrifugal force is replaced by a fluid stream in the axial direction. On the other hand, a reverse effect is observed for the revolving flow over a stationary disk, so that the fluid drawn to the axis of rotation is swept upwards, a phenomenon as a result of the radial pressure gradient being balanced by the centrifugal force. In the book by Schlichting [3] it was cited that "The secondary flow which accompanies rotation near a solid wall can be clearly observed in a tea cup: after the rotation has been generated by vigorous stirring and again after the flow has been left to itself for a short while, the radial inward flow field near the bottom will be formed. Its existence can be inferred from the fact that tea leaves settle in a little heap near the center at the bottom."

After the theoretical work of Bödewadt [1], the Bödewadt boundary layer was first observed on the flow on a rotor–stator system, see Batchelor [4]. The books by Owen and Rogers [5] and Shevchuk [6] give much significant information on flow and heat transfer in single rotating or double stationary/rotating systems and their applications. The recent studies of Sahoo et al. [7] and Sahoo and Sebastien [8] enlighten us about Bödewadt flow and heat transfer of a non-Newtonian Reiner–Rivlin fluid. The hydrodynamic stability of the flow produced over an infinite stationary plane in a fluid rotating with uniform angular velocity at an infinite distance from the plane was considered by Mackerrell [9] and Lopez et al. [10]. The possible occurrence of absolute instability mechanism was investigated by Jasmine and Gajjar [11].

The classical three-dimensional Von Karman viscous pump problem was recently extended by Fang [12] to the situation where the rotating disk is also radially stretching. The flow induced by a stretching boundary is known to be extensively important in the extrusion processes in plastic and metal industries [13,14]. Turkyilmazoglu [15–18] studied the linear and exponential radial stretching in magnetohydrodynamic rotating disk flows of traditional Von Karman. Such a treatment was also

* Tel.: +90 3122977850; fax: +90 3122972026.
E-mail address: turkyilm@hacettepe.edu.tr

http://dx.doi.org/10.1016/j.ijmecsci.2014.10.022
0020-7403/

implemented on the traditional Jeffery–Hamel flow in the very recent work [19].

In the literature the Bödewadt flow of a viscous fluid has not been at all elaborated when the stationary disk is set to a radial stretching. Hence, our motivation here is to fill this gap and generalize the classical Bödewadt flow under a stretchable wall condition. To serve to this aim, a spectral technique based on Chebyshev collocation is employed to numerically simulate the nonlinear partial differential equations of motion. Not only the flow but also the temperature field is mathematically analyzed. It is found that the radial stretch acts to cool down the wall, which is certainly important in engineering applications.

The rest of the paper is based on the subsequent strategy. The governing equations and their similarity counterparts are highlighted in Section 2. Section 3 discusses the results obtained. Conclusions are eventually drawn in Section 4.

2. Governing and similarity equations

The physical problem is formulated in the cylindrical coordinates (r, θ, z) such that the stationary wall is situated at $z=0$. The motion arises by the rotation of the fluid like a rigid body with constant angular velocity Ω at far distances from the disk surface. For reasons of axial symmetry the derivatives along the polar coordinate θ will be dropped. The flow field is represented by the vector $\mathbf{u}$ whose radial, tangential and axial velocity components are (u, v, w), respectively. The fluid is supposed to be Newtonian and viscous having the pressure p and density ρ. The fluid temperature is denoted by T such that the surface of the disk is maintained at a uniform temperature T_w. Far away from the wall, the revolving stream is kept at a constant temperature T_∞. It is also assumed that the fluid properties, viscosity (μ), thermal conductivity coefficient (κ), specific heat at constant pressure (c_p) and density (ρ) are all constants. The disk is further stretching at a uniform rate s in the radial direction r. Fig. 1 exhibits the flow description and geometrical coordinates. The governing Navier–Stokes and energy equations are then given by, see for instance [3,12,16],

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

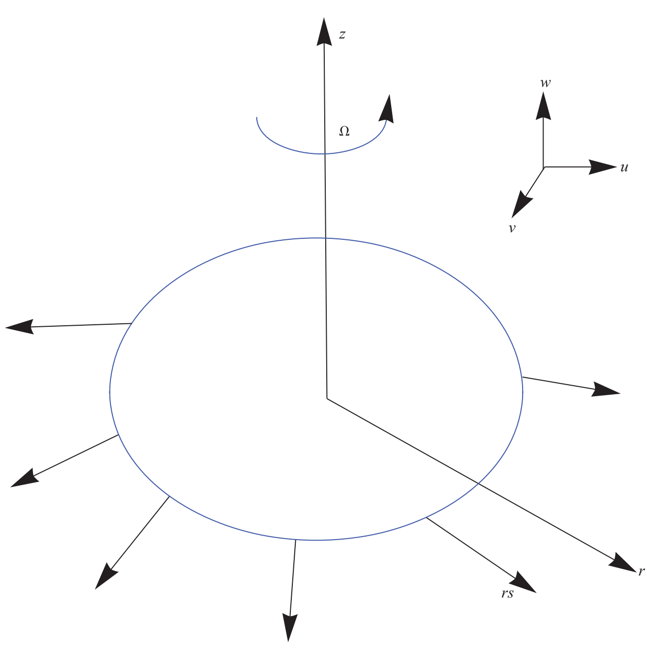

Fig. 1. Configuration of the flow and geometrical coordinates.

$$\rho(\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u}, \tag{2}$$

$$\rho c_p (\mathbf{u} \cdot \nabla) T = \kappa \nabla^2 T. \tag{3}$$

The boundary conditions accompanying (1)–(3) are

$$\begin{aligned} &u = sr, \quad v = 0, \quad w = 0, \quad T = T_w \quad \text{at } z = 0 \\ &u = 0, \quad v = \Omega r, \quad T = T_\infty \quad \text{as } z \to \infty, \end{aligned} \tag{4}$$

It is also known that the radial pressure gradient balances the centrifugal force at the frictionless regime, that is

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = r\Omega^2, \tag{5}$$

and in the framework of the boundary layer theory it is assumed that the viscous layer near the wall is also predominated by the action of the same pressure gradient.

By means of the dimensionless axial distance $\eta = (\sqrt{\Omega/\nu})z$, the form of similarity transformations is similar to those of conventional Von Karman flow [2], which are

$$\begin{aligned} &(u, v, w) = (r\Omega F(\eta), r\Omega G(\eta), \sqrt{\nu\Omega} H(\eta)), \\ &(p, T) = (p_\infty - \rho\nu\Omega P(\eta), T_\infty + (T_w - T_\infty)\theta(\eta)). \end{aligned} \tag{6}$$

Introducing Eqs. (5) and (6) into the governing equations (1)–(3), we obtain a system of ordinary differential equations that is nearly analogous to that stated in [16]:

$$\begin{aligned} &H' + 2F = 0, \\ &F'' - F^2 + G^2 - HF' - 1 = 0, \\ &G'' - 2FG - HG' = 0, \\ &\theta'' - PrH\theta' = 0. \end{aligned} \tag{7}$$

Here, a prime denotes derivative with respect to η and $Pr = \mu c_p / \kappa$ is the Prandtl number. The boundary conditions (4) are also converted to

$$\begin{aligned} &F - C = G = H = \theta - 1 = 0 \quad \text{at } \eta = 0 \\ &F = G - 1 = \theta = 0 \quad \text{as } \eta \to \infty, \end{aligned} \tag{8}$$

where $C = s/\Omega$ denotes a stretching strength parameter measuring the ratio of radial stretch to swirl such that $C=0$ corresponds to the classical non-stretching case. It should be remarked that Eqs. (7) and (8) differ from those given in [16] by the fact of the first momentum equation and also boundary conditions. After the principal solution had been obtained, the exact solution of the Navier–Stokes equations can be completed by calculating the pressure from the third momentum equation from (2), which is straightforward and hence omitted here.

Evaluation of the basic flow enables us to determine the skin friction at the wall as well as the heat transfer rate which are of practical importance. Hence, Newtonian formulae are made use for the tangential shear stress τ_θ and radial shear stress τ_r:

$$\begin{aligned} &\tau_\theta = \mu \left(\frac{\partial v}{\partial z}\right)_{z=0} = \mu R \Omega \sqrt{\frac{\Omega}{\nu}} G'(0), \\ &\tau_r = \mu \left(\frac{\partial u}{\partial z}\right)_{z=0} = \mu R \Omega \sqrt{\frac{\Omega}{\nu}} F'(0), \end{aligned} \tag{9}$$

which results in the total skin friction coefficient:

$$C_f = \frac{\sqrt{\tau_r^2 + \tau_\theta^2}}{\mu r \Omega \sqrt{\dfrac{\Omega}{\nu}}} = (F'(0)^2 + G'(0)^2). \tag{10}$$

Another physically interesting parameter is the total volume flowing towards the axis taken over a cylinder of radius R around the z-axis:

$$Q = 2\pi R \int_{z=0}^{\infty} u \, dz = -\pi R^2 \sqrt{\Omega\nu} H(\infty), \tag{11}$$