International Journal of Mechanical Sciences 90 (2015) 258–277

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Analytical sensitivity analysis of frequencies and modes for composite laminated structures

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ARTICLE INFO

Article history:
Received 30 July 2014
Received in revised form
3 November 2014
Accepted 23 November 2014
Available online 2 December 2014

Keywords:
Composite structures
Laminated plates
Finite element
Vibration
Analytical sensitivity

ABSTRACT

This paper describes an analytical method to calculate the sensitivity of the frequencies and modes with respect to fibre volume fractions and orientations, for the large-scale composite laminated structures with complex boundaries. Structural stiffness, mass matrices and their first derivatives with respect to fibre volume fractions and orientations are derived based on classical laminated plate theory and nonconforming rectangular element theory. The analytical sensitivity of the frequencies and modes with respect to fibre volume fractions and orientations is formulated based on vibration theory. Finally, the sensitivity analysis of the dynamic characteristics of T-shape and square composite laminated plates is demonstrated using the proposed analytical method. The accuracy and efficiency of the proposed method are also discussed. The proposed method can be employed to analyze the sensitivity of the frequencies and modes for the composite laminated structures.

1. Introduction

Composite laminated structures are gaining more and more applications in aircraft, automobile, naval and defence industries because of their attractive performance characteristics, such as high strength-to-weight ratio, high stiffness-to-weight ratio, superior fatigue properties and high corrosion resistance [1,2]. These composite laminated structures are often working in the vibration environments. Therefore, considering the dynamic characteristics and responses in optimization design is very important. Sensitivity analysis is often employed to determine the searching direction in structural optimization design and to modify the design variables in structural modification design. Sensitivity analysis technique has become a critical technique for the gradient-based optimization methods and structural modification design. Many works on the static sensitivity analyses for the structures, made of homogeneous and isotropic material, have been reported [3]. For examples, the analytical and semi-analytical sensitivity analysis methods using the hybrid finite elements [4–6] and semi-analytical shape sensitivity analysis methods based on numerical differentiation of the finite element matrices [7,8]. The dynamic sensitivity analysis methods for the engineering structures, made of homogeneous and isotropic material, have also been studied widely [9]. For examples, sensitivity analysis methods for eigenvalues and eigenvectors [10–12], sensitivity analysis for transient responses of structures subjected to time history loading [13–17] and sensitivity analysis for random responses of structures subjected to stochastic ground motion [18,19]. The options of structural design sensitivity analysis methods including static and transient responses for linear structure systems have been reviewed in detail by Van Keulen et al. [20]. However, the literatures on dynamic sensitivity analysis for the composite laminated structures are very limited. Grenestedt [21] employed both numerically by finite difference calculations and analytically by a perturbation approach to calculate the sensitivity of the lowest vibration frequency and to find the layup that maximizes the lowest free vibration frequency of the classical laminates. The analytical expressions are available for the simply supported orthotropic plates. Mota Soares et al. [22] studied the sensitivity analysis for the optimization of the multi-layered composite axisymmetric shells subjected to the arbitrary static loading and free vibrations. The sensitivities are calculated analytically, semi-analytically and by global finite difference. Zabihollah et al. [23] conducted the sensitivity analysis of laminated beams with surface bonded and/or embedded piezoelectric sensors and actuators using the finite element model based on the layerwise displacement theory. Li et al. [24] employed the overall finite difference method and the semi-analytical method to calculate the sensitivity coefficients of displacements, stresses and frequencies of the composite sandwich plates based on a layerwise/solid-element method. Dems [25,26] performed the sensitivity analysis with respect to material parameters and fibres' shape for the fibre reinforced composite disks. Liu et al. [27] developed a mapping method for sensitivity analysis of composite material property. Li and Qing [28] employed the meshless method for the sensitivity analysis of composite laminated

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http://dx.doi.org/10.1016/j.ijmecsci.2014.11.018
0020-7403/

plates in the state space framework. The sensitivity analysis methods can be divided into three categories: finite-difference methods, analytical methods and semi-analytical methods [29]. The finite-difference methods are the easy implemented sensitivity analysis methods. Compared with analytical methods and semi-analytical methods, the finite-difference methods are often not efficient and accurate. The analytical methods and semi-analytical methods are often employed to analyze the sensitivity of composite laminated structures with regular shape and simple boundary conditions.

The purpose of this paper is to develop an analytical sensitivity analysis method to calculate the first derivatives of frequencies and modes with respect to fibre volume fractions and fibre orientations, for the large-scale composite laminated structures with complex boundaries. The proposed analytical sensitivity analysis method is based on the classical laminated plate theory, nonconforming rectangular element theory and vibration theory. In Sections 2–5, the total stiffness, mass matrices and their first derivatives of composite laminated structures, with respect to fibre volume fractions and fibre orientations, are derived based on the classical laminated plate theory and nonconforming rectangular element theory. In Section 6, the first derivatives of frequencies and modes of composite laminated structures, with respect to fibre volume fractions and fibre orientations, are formulated using vibration theory. Finally, the sensitivity analysis of frequencies and modes for T-shape and square composite laminated plates are demonstrated using the proposed analytical sensitivity analysis method. The accuracy, efficiency and applications of the proposed method are also discussed.

2. First derivatives of extensional stiffness, bending stiffness and bending-extensional coupling stiffness with respect to fibre volume fraction or fibre orientations

A rectangular element of laminate shown in Fig. 1 is cut out from a composite laminated structure. The thickness of laminate is h. The laminate element coordinate system is xyz and the laminate element number is e. The four local nodal numbers and corresponding coordinates are $1(-a,-b)$, $2(a,-b)$, $3(a,b)$, and $4(-a,b)$. The laminate is made of N layers. The number of layers are denoted by (1), (2), ⋯, (N) and the fibre orientations of layers are denoted by $\theta^{(1)}, \theta^{(2)}, \cdots, \theta^{(N)}$. The layered positions are denoted by $z_1, z_2, \cdots, z_N, z_{N+1}$.

The lth layer of laminate called lamina is shown in Fig. 2. The material coordinate system is $x_1x_2x_3$. The angle $\theta^{(l)}$ where $(-90^\circ \le \theta^{(l)} \le 90^\circ)$ between axes x and x_1 indicates the fibre orientation in the lth lamina.

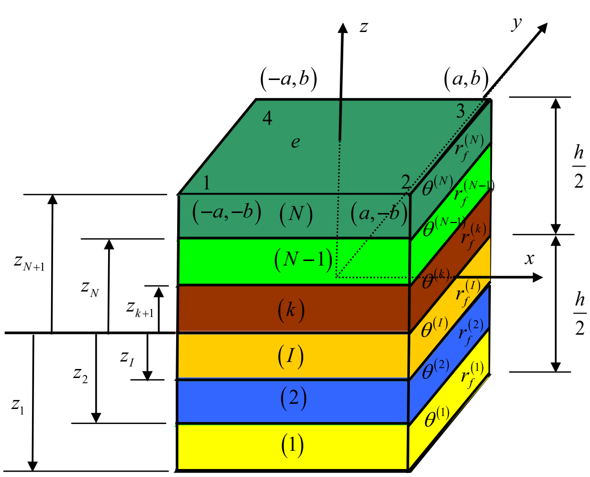

Fig. 1. A rectangular element of laminate ($\theta^{(l)}$ and $r_f^{(l)}$ are design variables, where $l=1,2,\cdots,N-1,N$).

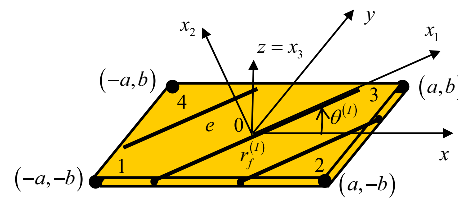

Fig. 2. The lth layer of laminate ($\theta^{(l)}$ and $r_f^{(l)}$ are design variables).

2.1. Constitutive relations of lamina

In the classical laminated plate theory [1], the linear constitutive relations for the lth orthotropic lamina in the material coordinate system $x_1y_1z_1$ are as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}^{(l)} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}^{(l)} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}^{(l)} \tag{1}$$

where the plane stiffness coefficients $Q_{ij}^{(l)}$ of the lth lamina are as follows:

$$Q_{11}^{(l)} = \frac{E_1}{1-\nu_{12}\nu_{21}},\quad Q_{12}^{(l)} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}},$$

$$Q_{22}^{(l)} = \frac{E_2}{1-\nu_{12}\nu_{21}},\quad Q_{66}^{(l)} = G_{12} \tag{2}$$

where E_1 is the longitudinal modulus, E_2 is the transverse modulus, ν_{12} and ν_{21} are the Poisson's ratios and G_{12} is the shear modulus, of the lamina. The engineering constants (E_1, E_2, $\nu_{12}=\nu_{21}$ and ν_{21}) can be determined as follows by using micromechanics approach. The detailed derivation is given in Chapter 3.2 of Jones [30].

$$E_1 = E_f r_f^{(l)} + E_m r_m^{(l)} = E_f r_f^{(l)} + E_m\left(1-r_f^{(l)}\right) \tag{3}$$

$$E_2 = \frac{E_f E_m}{E_f r_m^{(l)} + E_m r_f^{(l)}} = \frac{E_f E_m}{E_f\left(1-r_f^{(l)}\right) + E_m r_f^{(l)}} \tag{4}$$

$$\nu_{12} = \nu_{21} = \nu_f r_f^{(l)} + \nu_m r_m^{(l)} = \nu_f r_f^{(l)} + \nu_m\left(1-r_f^{(l)}\right) \tag{5}$$

$$G_{12} = \frac{G_f G_m}{G_f r_m^{(l)} + G_m r_f^{(l)}} = \frac{G_f G_m}{G_f\left(1-r_f^{(l)}\right) + G_m r_f^{(l)}} \tag{6}$$

where

$$G_f = \frac{E_f}{2(1+v_f)},\quad G_m = \frac{E_m}{2(1+v_m)} \tag{7}$$

and E_f is the modulus of elasticity of the fibre, E_m is the modulus of elasticity of the matrix. ν_f is Poisson's ratio of the fibre and ν_m is Poisson's ratio of the matrix. $r_f^{(l)}$ and $r_m^{(l)}$ are the fibre volume fraction and matrix volume fraction of the lth lamina, respectively. The plane stiffness coefficient $Q_{ij}^{(l)}$ of the lth lamina is a function of the fibre volume fraction $r_f^{(l)}$ of the lth lamina.

The linear constitutive relations for the lth orthotropic lamina in the laminate element coordinate system xyz are as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^{(l)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}^{(l)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^{(l)} \tag{8}$$