International Journal of Mechanical Sciences 105 (2016) 70–82

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Numerical and analytical solutions for anisotropic yield surfaces of the open-cell KELVIN foam

Johannes Storm, Martin Abendroth, Meinhard Kuna *

TU Bergakademie Freiberg, Institute for Mechanics and Fluid Dynamics, Lampadiusstraße 4, 09596 Freiberg, Germany

ARTICLE INFO

Article history:
Received 11 May 2015
Received in revised form
16 October 2015
Accepted 25 October 2015
Available online 12 November 2015

Keywords:
Open-cell foam
KELVIN cell
Yield surface
Homogenisation

ABSTRACT

An analytical solution for the effective yield surface of the KELVIN cell is presented. Several local strength and yield criteria are applied like VON MISES, maximum principal normal stress with and without tension/compression asymmetry. These criteria are representative for the yielding or failure of different ductile or brittle materials. Their influence on the effective failure behaviour of foams is investigated. An efficient numerical method is derived to determine anisotropic initial and subsequent yield surfaces for open-cell foams. It is applied to the KELVIN cell model and used to verify the analytical solution. The results are used to determine the conservative, isotropic minimum yield surface for the KELVIN cell. The properties of the different effective yield surfaces are evaluated and discussed.

1. Introduction

Cellular materials are more and more used as engineering material. There are manufacturing methods, which allow the deliberate manipulation of the effective material behaviour by changing the foam morphology, e.g. selective laser sintering. Foamed components can be analysed on different length scales: micro-scale (bulk material), meso-scale (foam topology and morphology) and the macro-scale (effective component behaviour). The homogenised, effective behaviour of course depends on the bulk material but also strongly on the foam structure, e.g. pore shape, cell walls, and strut forms [5].

An essential engineering task is to describe the effective component behaviour. The computational effort for simulations, which contain two or more length scales, is huge. Such simulations are strongly restricted by the computing power and main memory. There are methods for multi-scale simulations, which reduce the computational effort, like FE^2. Another method is to completely separate the scales and to determine an effective material behaviour for the smaller scales using homogenisation, which gives the constitutive equations for the corresponding parent scales. The homogenisation method in mechanics can be used if the stress and strain fields can be separated on the different scales (scale separation [12]). The characteristic length of the mesostructural fluctuations has to be smaller than the representative volume of the mesostructure, which must be smaller than the characteristic length of the macrostructural fluctuations. For strong stress and strain gradients on the macro-scale the separation conditions can be violated and have to be checked.

The determination and evaluation of effective elastic constants of cellular materials has been well investigated [30,26,29,10,20,19], whereas the nonlinear material response is still subject of current research. Performing experiments to determine failure and yield surfaces is challenging. Some results are presented for example in Thornton and Magee [23], Gibson and Ashby [9], Triantafillou et al. [25], Triantafillou and Gibson [24], Deshpande and Fleck [7], Degischer and Kriszt [5], Combaz et al. [3]. There are also models for the failure surfaces of brittle cellular materials, e.g. Triantafillou et al. [25], Theocaris [21], Hanssen et al. [11], Piccolroaz and Bigoni [18], Bartl [2], and Daxner [4]. Many simulation approaches of foam structures for strongly nonlinear, path dependent behaviour make use of the nested finite element method FE^2 [14,17]. In this kind of multi-scale simulation the macro-scale simulation calls a corresponding meso-scale model at every GAUSSIAN point assigning the current local deformation state. The material response is thus determined by the underlying meso-scale simulation, and the full path-dependent material behaviour can be captured. Especially, for complex macroscopic material behaviour, the FE^2 method can overcome the challenges to provide a closed form constitutive model for the macro-scale. But this method requires huge computational power due to the number of meso-scale simulations and can be succeeded only by parallel high performance computations.

The other approach is to identify constitutive equations by performing classical homogenisation of the meso-scale. Once this effective material law is obtained for a specific kind of foam the

* Corresponding author. Tel.: +49 3731 392092.
E-mail address: meinhard.kuna@imfd.tu-freiberg.de (M. Kuna).

http://dx.doi.org/10.1016/j.ijmecsci.2015.10.014
0020-7403/

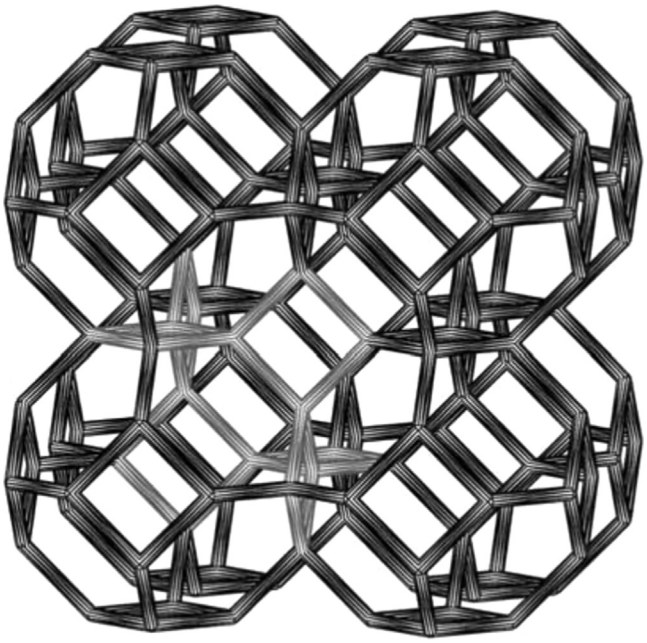

Fig. 1. Kelvin foam with highlighted characteristic Kelvin unit cell [10].

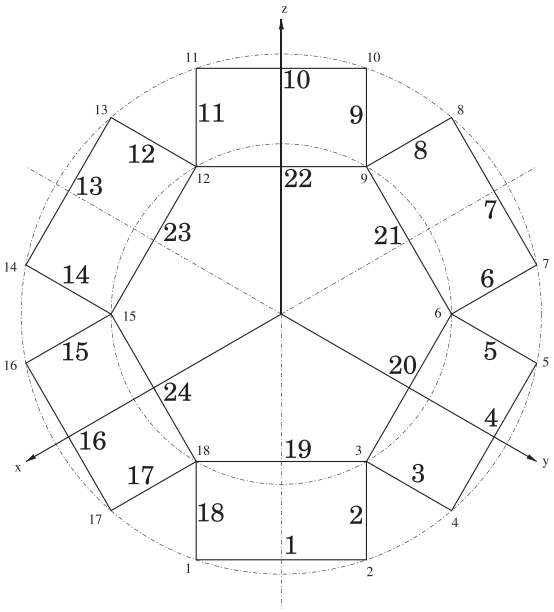

Fig. 2. Parallel octahedral plane projection of a characteristic unit cell of a periodic Kelvin foam seen from point [1,1,1].

simulation on the macro-scale is an ordinary one-scale simulation and there are no special hardware requirements. The effective elastic–plastic material law requires to determine the effective elastic constants, but also the yield surface and the yield surface evolution by means of meso-scale simulations. Analyses of the yield surface and subsequent yield surfaces of foams are presented e.g. in Laroussi et al. [15], Wicklein and Thoma [28], Demiray [6], and Daxner [4]. The classical homogenisation method is used to obtain the relation between effective stresses and strains. The procedure presented in these papers to localise the yield stress along a load path requires several elastic–plastic simulations steps. The deviation of the result from the exact value depends on the number of these steps. Furthermore, the resolution of the yield surfaces depends on the number of analysed load paths. The presented results are determined for a specific choice of effective principal axes, which means the anisotropy of the yield surfaces is neglected.

In this work we analyse the limits of the linear elastic domain of the open-cell Kelvin foam. This can be the failure surface for brittle ceramic foams or the yield surface for foams with elastic–plastic bulk material. Moreover, the anisotropy of this surface is investigated and its importance for conservative component design is shown by comparison with the isotropic minimum surface. Finally, an analytical solution for the anisotropic yield surface of the Kelvin cell is presented and the influence of several yield and strength criteria is analysed.

The presented numerical method to determine the effective, initial and subsequent yield surfaces is based on classical homogenisation theory using representative volume elements (RVE) and the assumption of a linear elastic domain on the meso- and macro-scale. Each effective, anisotropic initial yield surface can be determined from six independent load cases of a linear-elastic simulation. One nonlinear simulation and six linear simulations are sufficient for every subsequent yield surface. For nonlinear processes with small deformations and if the elastic properties of the bulk material remain the same the six linear simulations can be performed once for all subsequent yield surfaces. Thus, the resolution of the yield surface is decoupled from the number of required simulations. The assembly of the yield surface requires a more complex post-processing, but with much less computational effort than solving the boundary value problem of the RVE. In the current paper the method is applied to a beam model of the Kelvin cell, which allows to determine the minimum surface in reasonable time. But the method is further the basis to determine the yield and failure surface of volumetric models and stochastic foam structures, which is the content of an upcoming publication of the authors. The huge computational effort needed for such models makes it inexpedient to apply methods previously presented in the literature [6,4]

The method has the potential to improve the performance of FE^2 simulations. Because the check for an elastic step and the calculation of the elastic result using the presented method are computationally cheap the performance of any linear-elastic step of the macroscopic Gaussian points is in the same order as for the classical FEM. For a recognised nonlinear step the classical procedure of the FE^2 method is used.

2. Model description

2.1. Topology of the Kelvin cell

The Kelvin cell was first introduced by Lord Kelvin (Sir William Thomson) [22,27] when he was studying soap froths. It is a simple periodic cell structure with very low surface energy. The structure of the so-called *characteristic* Kelvin *unit cell* is used as RVE of the Kelvin foam and shown in Figs. 1 and 2. All struts have a length of l_0, thus the entire cell model length l is $2\sqrt{2}\,l_0$. The struts (and walls) of natural Kelvin cells are slightly curved and the cross-section area of the struts varies along the strut axes. These details have been shown to be important for mechanical models but are often neglected for the sake of simplification [20,19].

The Kelvin cell has octahedral symmetry. It is mechanically cubic orthotropic. The anisotropy factor depends on the strut geometry. A coordinate system is introduced with its origin placed at the centre of this RVE (hexagon) and the axes intersect the RVE boundaries perpendicularly.