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Stresses and moments in through-thickness functionally graded plate weakened by circular/elliptical cut-out

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ABSTRACT

In this paper, solution of stress resultant and moments around circular and elliptical hole in functionally graded (FG) infinite plate with through thickness material property variation is presented. The through thickness variation of material properties is obtained according to power law and stress functions are computed using complex variable method. Effect of gradation of material property, geometry and loading angle on stresses and moments is studied.

1. Introduction

It is important to study stress and moment concentration around the hole in infinite plate when it is subjected loading at infinity. The hole in the plate may be due to design/operational requirement or any damage like corrosion, wear or accident. Researchers have been contributing to establish solution technique to calculate stress and moment concentration around hole of different shape in infinite plate of isotropic and anisotropic material since long.

Muskhelishvili [1] has developed complex variable method to solve the problems of 2D theory of elasticity. Savin [2] and Lekhnitskii [3] were among the first few persons who contributed significantly in solving problems of stress analysis of isotropic and anisotropic plates with holes (circle, ellipse, square, rectangle, triangle) using Muskhelishvili [1] method. Simha and Mohapatra [4] (irregular shaped hole), Rezaeepazhand and Jafari [5] (triangle, square, polygonal shaped hole), Gao [6] (elliptical hole), Sharma [7,8] (polygonal shaped hole) have presented solution in an infinite isotropic plate and Daoust and Hoa [9] (triangular shaped hole), Sharma [10,11] (circular, elliptical, triangular, polygonal shaped hole), Sharma and Dave [12] (hypocycloidal shaped hole), Sharma and Patel [13] (square shaped hole), Chauhan and Sharma [14] etc. have given solution in an infinite anisotropic plate for stresses around holes of different shapes using complex potentials. The above mentioned references primarily focused either an isotropic plate or symmetric anisotropic plates. The coupling stiffness exists in unsymmetrical laminates. Backer [15,16] (elliptical shaped hole), Ukadgaoker and Rao [17] (circular, elliptical, square, rectangular shaped hole etc.) and Chen and Shen [18,19] (circular, elliptical shaped hole) addressed the problem of stress and moment concentration around hole in an infinite unsymmetrical laminates.

Functionally graded (FG) materials are comparatively new comer in the domain of composite material and may prove itself advantageous over conventional composite material. In structural application such FG plate may have some holes which may lead towards the stress and moment concentration and it should be addressed effectively. The problem of stress concentration around circular hole in FG plate has been addressed by Chen [20,21], Mohammadi and Dryden [22], Mohammadi, Dryden and Jiang [23], Kim and Paulino [24], Kubair and Bhanu-Chandar [25] and Enab [26]. They have taken material property variation in radial direction. The property of the FG plate can also vary through thickness using some power law. Batra and Jin [27], Pan [28], Ramirez, Heyligera and Pan [29], Croce and Venni [30] and Reid and Pasakarmoorthy [31] have used through thickness variation of the properties and solved different problems. The through thickness variation of the properties may bring in un-symmetry and therefore coupling stiffness may exists. To the best of author's knowledge, stress and moment distribution around elliptical hole in functionally graded infinite plate with through thickness property variation is not yet addressed.

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Nomenclature

A_5, A_6, A_7, A_8	complex constants
E_1	Young's Modulus along axis 1
E_2	Young's modulus along axis 2
E_x	Young's Modulus along axis X
E_y	Young's Modulus along axis Y
G_{12}	shear modulus for plane 1–2
G_{xy}	shear modulus for plane X–Y
H	total thickness
M_x, M_y, M_{xy}	moment resultant in FG plate
$MP(z)$	normalized material property at layer height z
N_x, N_y, N_{xy}	stress resultant in the FG plate
P_t	material property at the top of the FG plate
P_b	material property at the bottom of the FG plate.
$P(z)$	material property at layer height z
Q_x, Q_y	components of shear force
Q	shear force
R	constant of mapping function
S'	stress resultant at infinity
U	stress function
aa	semi-major axis
bb	semi-minor axis
a_j, b_j	constants in mapping function
e_{1j}, e_{2j}, e_{3j}	complex constants for mid plane strain
f_{1j}, f_{2j}, f_{3j}	complex constants for moments
m	constant of mapping function
$m(s)$	normal bending moment on hole contour
p	index for gradation of material properties
p_j, q_j	complex constants of laminate
$p(s)$	transversal force on hole contour
s_j	complex parameter of anisotropy, $j=1$–4
u, v, and w	in-plane and normal displacement
h	layer height from central plane
Φ_{1j}	stress function of plate without subjected to loading at infinity
α	polar angle
β	Loading angle
λ	biaxial loading factor
υ_{12}	Poisson's ratio for plane 1–2
υ_{xy}	Poisson's ratio for plane X–Y
$\xi=e^{i\alpha}$	unit circle
$\varpi(\xi)$	mapping function in ξ plane
ϕ_{2j}	stress function for plate with hole and absence of loading
ϕ_j	final stress function
$\phi_j'(z_j), \phi_j''(z_j), \phi_j'''(z_j)$	First, second and third derivatives of stress function

The purpose of this paper is to provide solution of stresses and moment around circular and elliptical hole in functionally graded infinite plate with material property variation along the thickness using complex variable approach. The mathematical formulation is presented to find stress resultant and moments around hole geometry and effect of gradation of material property, size of hole and loading conditions on stress and moment distribution is studied.

2. Analytical formulations

A functionally graded infinite plate with a hole is shown in Fig. 1. The origin of global coordinate system is taken at mid plane of the plate and at the centre of the hole. The material axes 1–2–3 has its origin at the centre of hole and h distance away from the mid plane. It is to be noted here that Z-axis and 3-axis are coincident and plane X–Y and 1–2 are parallel to each other. Material axes are aligned with global coordinate axes at the top and orthogonal to global coordinate axes at the bottom.

The Young's modulus, shear modulus and Poisson's ratio can be obtained at location h along the thickness as per power law given as,

$$MP(h)=\frac{P(h)}{P_t}=1+\left(\frac{P_b}{P_t}-1\right)\left(\frac{1}{2}+\frac{h}{H}\right)^p, P_t\neq P_b. \tag{1}$$

where, h=distance from XY plane, $MP(h)$=Normalized material property at h distance away, H=total thickness of the plate, p=index for gradation of material properties, $P(h)$=Material property at distance h, P_t=Material property at the top of the FG plate, P_b=Material property at the bottom of the FG plate.

The angle of orientation (θ) of material axes with respect to global axes can be obtained such that the value of off axes engineering constants varies as per power law (Eq. (1)).

The FG plate has stress resultants at infinity along coordinate system $X'-Y'$, oriented arbitrarily at an angle β with X–Y coordinate system. The stress resultant along X–Y can be evaluated from Gao's [6] biaxial loading condition as given below,

$$N_x^\infty+N_y^\infty=S'(\lambda+1),$$

$$N_y^\infty-N_x^\infty+2iN_{xy}^\infty=S'e^{-2i\beta}(1-\lambda). \tag{2}$$

where, λ is biaxial loading factor and S' is stress resultant at infinity.

Let $U(x,y)$ be the stress function (Airy's stress function). The stress resultants in the absence of body forces can be written as,

$$N_x=\frac{\partial^2U}{\partial y^2}, N_y=\frac{\partial^2U}{\partial x^2} \textit{ and } N_{xy}=-\frac{\partial^2U}{\partial x\partial y}, \tag{3}$$

which satisfy equations of equilibrium and compatibility conditions.

The stresses resultants $N=\begin{bmatrix}N_x & N_y & N_{xy}\end{bmatrix}^T$ and plate moments $M=\begin{bmatrix}M_x & M_y & M_{xy}\end{bmatrix}^T$ in FG plate can be related with mid plane strain $\epsilon^0=\begin{bmatrix}\epsilon_x^0 & \epsilon_y^0 & \gamma_{xy}^0\end{bmatrix}^T$ and curvature $k=\begin{bmatrix}k_x & k_y & k_{xy}\end{bmatrix}^T$ using generalized Hooke's law as,

$$\begin{Bmatrix}N_x\\N_y\\N_{xy}\\M_x\\M_y\\M_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11}&A_{12}&A_{16}&B_{11}&B_{12}&B_{16}\\A_{12}&A_{22}&A_{26}&B_{12}&B_{22}&B_{26}\\A_{16}&A_{26}&A_{66}&B_{16}&B_{26}&B_{66}\\B_{11}&B_{12}&B_{16}&D_{11}&D_{12}&D_{16}\\B_{12}&B_{22}&B_{26}&D_{12}&D_{22}&D_{26}\\B_{16}&B_{26}&B_{66}&D_{16}&D_{26}&D_{66}\end{bmatrix}\begin{Bmatrix}\epsilon_x^0\\\epsilon_y^0\\\gamma_{xy}^0\\k_x\\k_y\\k_{xy}\end{Bmatrix} \tag{4}$$

where, $[A_{ij}]_{3X3}$, $[B_{ij}]_{3X3}$ and $[D_{ij}]_{3X3}$ are extensional, coupling and bending stiffness respectively. The mid plane strain and curvature is written in terms of displacement u_0 and v_0 in X–Y plane and out of plane displacement w as,

$$\begin{bmatrix}\epsilon_x^0 & \epsilon_y^0 & \gamma_{xy}^0\end{bmatrix}^T=\begin{bmatrix}\frac{\partial u_0}{\partial x} & \frac{\partial v_0}{\partial y} & \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{bmatrix}^T \tag{5}$$

and

$$\begin{bmatrix}k_x & k_y & k_{xy}\end{bmatrix}^T=-\begin{bmatrix}\frac{\partial^2 w}{\partial x^2} & \frac{\partial^2 w}{\partial y^2} & 2\frac{\partial^2 w}{\partial x\partial y}\end{bmatrix}^T. \tag{6}$$