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Stress analysis of thick pressure vessel composed of functionally graded incompressible hyperelastic materials

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ABSTRACT

The present paper develops stress analysis of functionally graded hyperelastic thick spherical shell subjected to internal and external pressure. Hyperelastic behavior is modeled by using modified neo-Hookean strain energy function with variable material parameters. The material constants of strain energy function are graded along the radial direction based on a power law function. Material inhomogeneity parameter (m) is a power in the mentioned power law function. Material constant of strain energy function calculated from experimental data by using Levenberg-Marquardt nonlinear regression method. Stress components and stretches of spherical shell have been obtained for axisymmetric radial condition. Following this, profiles of extension ratio and stress components are plotted as a function of radius of sphere in the undeformed configuration for different material inhomogeneity parameter (m). The obtained results show that the inhomogeneity properties of FGMs have a significant influence on the displacement, stretch and stresses distribution along the radial direction. Sensitivity of stress components and displacement field to applied pressure and outer to inner radius ratio of vessel (structure parameter) are also investigated and leads to the fact that outer to inner radius ratio has a great effect on the deformation field and stress components and is a useful parameter from a design point of view which can be tailored to specific applications to control the stress. Thus with selecting a proper m value and structure parameter (B/A), engineers can design a specific FGM hollow sphere that can meet some special requirements.

1. Introduction

The principal problem in the elasticity theory is to find the relation between the stress and the strain in a body under certain external forces. In small deformation, linear elasticity and Hooke's law are applied to find stress–strain relation but in large deformation, materials show nonlinear elastic behavior which can be characterized by hyperelasticity. Rubber and rubber like materials are assumed incompressible hyperelastic materials. Because of specific application characteristics and economic advantages of rubber and rubber like materials, the corresponding structures, such as tubes, rings, shells, spheres and pads, composed of these materials, are widely used in petrochemical, aerospace, biomedical and many other fields of human life. Simple constitutive relations for studying their mechanical behavior include the neo-Hookean and the Mooney–Rivlin strain energy functions. Modeling the mechanical behavior of rubber like materials within the framework of nonlinear elasticity theory was the subject of intense investigations, which could be found in the review articles contributed by Beatty [1], Horgan and Polignone [2], Attard [3] and the monograph contributed by Fu and Ogden [4]. In recent years several researches were done on constitutive modeling of rubber like materials such as works by Anani and Alizadeh [5], Tomita et al. [6], Coelho et al. [7] and Santos et al. [8].

Shells are general structural elements in many engineering applications, including pressure vessels, sub-marine hulls, ship hulls, wings and fuselages of airplanes, automobile tires pipes, exteriors of rockets, missiles, concrete roofs, chimneys, cooling towers, liquid storage tanks, and many other structures. Furthermore they are found in nature in the form of leaves, eggs, inner ear, bladder, blood vessel, skulls, and geological formations [9], therefore, the main concern of this paper is thick spherical shell made of FG rubber like materials. Rapid growth in technology has ushered in an era when it is possible to synthesize materials that exhibit a variation/graded-variation in their properties for proper design in different components such as shells and pressure vessels. An FGM is nonhomogeneous in composition. Inhomogeneities can be introduced in rubber like materials either during vulcanization or uneven interaction with thermal, radiative and oxidative

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environments [10]. The properties of these materials may change gradually and continuously along the coordinate. These variable properties can be simulated using an analytical function. This functional distribution of material properties can help researchers in order to control the distribution of displacement or stress in a solid structure. There is extensive literature on FGMs in linear elasticity and it is impossible to review it in a paper other than in a review article but some significant researches are cited here. Obata and Noda [11] through the application of a perturbation approach investigated the thermal stresses in an FGM hollow sphere and in a hollow circular cylinder. The aim of their study was to find out the effect of composition on stresses and to design the optimum FGM hollow circular cylinder and hollow sphere. Closed-form solutions for cylindrical and spherical vessels with variable elastic properties were obtained by Tutuncu and Ozturk [12]. They modeled the variable material properties with simple power law function through the wall thickness which resulted in simple Euler–Cauchy equations whose solutions were readily available. You et al. [13] analyzed elastic behavior of internally pressurized thick walled spherical pressure vessels of functionally graded materials. Numerical method was used by Chen and Lin [14] to find stresses and displacements in spherical and cylindrical FG pressure vessels with constant Poisson ratio and exponential variation in radial direction for the modulus of elasticity. A hollow sphere made of FGMs subjected to radial pressure was analyzed by Li et al. [15]. They studied a hollow sphere made of FGMs subjected to radial pressure and then they reduced the problem to a Fredholm integral equation. Axisymmetric displacements and stresses in functionally graded hollow cylinders, disks and spheres subjected to uniform internal pressure were studied by Tutuncu and Temel [16]. They used plane elasticity theory and complementary functions method in their research. Zamaninejad et al. [17] developed a 3-D set of field equations of FGM thick shells of revolution in curvilinear coordinate system by tensor calculus. They assumed material properties vary nonlinearly in the radial direction, and Poisson ratio is constant, then exact solutions for stresses and displacement in a functionally graded pressurized thick-walled hollow circular cylinder are obtained under generalized plane strain and plane stress assumptions, respectively. Borisov [18] studied deformations and stresses inside multilayered thick-walled spheres. Each sphere is characterized by its elastic modules. The zone of contact between each of the spheres is continuous on the surface. In another study material tailoring for functionally graded linear elastic hollow cylinders and spheres was done by Nie et al. [19] to obtain either a constant circumferential stress or a constant in-plane shear stress through the thickness. They supposed the volume fractions of two phases of an FG material (FGM) vary only with the radius. Ghannad and Zamaninejad [20] derived the governing equations for axisymmetric thick spherical shells made of nonhomogeneous functionally graded materials subjected to internal and external pressure in the general case. They assumed constant Poisson ratio and power law variation for modulus of elasticity in the radial direction.

In spite of numerous works on FGMs in linear elasticity, there is hardly any research related to functionally graded rubber like materials. For the first time, graded rubber like materials were created by Ikeda [21] via a construction-based method in the laboratory. Bilgili et al. [22] also treated the material nonhomogeneity within the context of hyperelasticity and finite thermoelasticity. Their studies demonstrated that the spatial variation pattern of the nonhomogeneity has a deep influence on the stress-strain fields and that strong localization in the form of boundary layers can occur. Moreover, they worked to illuminate effects of material inhomogeneities on through-the-thickness stress distributions and how to use them for optimally designing circular cylinders made of rubber like materials. In another study plane strain deformations of a circular cylinder made of an inhomogeneous neo-Hookean material with circumferential displacements prescribed on the inner and the outer surfaces studied by Bilgili [23]. Functional grading of rubber tubes within the context of a molecularly inspired finite thermoelastic model is subject of another research by Bilgili [24]. As an application, the azimuthal shearing of a hollow rubber tube subjected to thermal loading was considered with a view toward minimizing the strain inhomogeneity, and an optimum grading for each temperature gradient was presented. Role of inhomogeneities in the deformation of elastic bodies was studied by Saravanan and Rajagopal [25]. In another study a comparison of the response of isotropic inhomogeneous elastic cylindrical and spherical shells and their homogenized counterparts was done by Saravanan and Rajagopal [26]. The same authors analyzed inflation, extension, torsion and shearing of an inhomogeneous compressible elastic right circular annular cylinder [27]. Iaccarino and Batra [28] analyzed radial expansion/contraction of a hollow sphere composed of a second-order elastic, isotropic, incompressible and inhomogeneous material with the two material parameters smoothly varying in the radial direction to delineate differences and similarities between solutions of the first- and the second-order problems. Batra [29] analyzed the torsion of a cylinder made of incompressible Hookean material with the shear modulus varying along the axial direction, and found the axial variation of the shear modulus to manage the angle of twist of a cross-section. Batra and Bahrami [30] investigated cylindrical pressure vessel made of FG rubber like material under internal pressure. To find stress components of the pressure vessel, they assumed axisymmetric radial deformations of a circular cylinder composed of FG Mooney–Rivlin material with the material parameters varying continuously through the radial direction either by a power law or an affine relation. They found that for the exponent of the power law function equal to 1, the hoop stress for an internally pressurized cylinder is uniform in the cylinder. In another study, material tailoring and universal relations for axisymmetric deformations of functionally graded rubberlike cylinders and spheres was presented by Batra [31].

Surveying through the researches done on different structures gives a clear view of a gap in analyzing spherical shell made of FG rubber like materials although such spherical shells gain attentions in engineering applications. Therefore, stress analyzing of FG rubber like material spheres is the objective of this research. To that end, exact analytical solution is derived for stresses and displacements of pressurized thick spheres made of isotropic functionally graded rubber like materials (FGM) with power law variation material properties in radial direction.

2. Hyperelasticity

Hyperelasticity is the study of nonlinear elastic materials undergoing large deformation. Rubbers are typically hyperelastic. In hyperelasticity, the stress is not calculated directly from strain as in the case of small strains, linear elastic materials. Instead, stresses are derived from the principle of virtual work using the stored strain energy potential function W, which is expressed with principal invariants of deformation gradient tensor $\boldsymbol{F}$. The deformation gradient $\boldsymbol{F}$ relates quantities before deformations to them after or during deformations. Consider a point at position $\boldsymbol{X}$. If this point is displaced to a new position $\boldsymbol{x}$, then the deformation gradient tensor $\boldsymbol{F}$ is defined as:

$$\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}. \tag{1}$$

The deformation gradient tensor can be decomposed into stretch and rotation parts using polar decomposition: $\boldsymbol{F} = \boldsymbol{VR} = \boldsymbol{RU}$, where