International Journal of Mechanical Sciences 104 (2015) 60–74

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

A variational Ritz formulation for vibration analysis of thick quadrilateral laminated plates

Rita F. Rango *, Facundo J. Bellomo, Liz G. Nallim

INIQUI – CONICET – Aula CIMNE – Facultad de Ingeniería – Universidad Nacional de Salta, Av. Bolivia 5150, 4400 Salta, Argentina

ARTICLE INFO

Article history:
Received 15 July 2015
Received in revised form
22 September 2015
Accepted 29 September 2015
Available online 22 October 2015

Keywords:
Thick plates
Quadrilateral plates
Ritz method
Free vibration

ABSTRACT

In the present study, a variational Ritz approach for vibration analysis of thick arbitrarily quadrilateral laminated plates, based on the trigonometric shear deformation theory (TSDT) is developed. In this theory, shear stresses are vanished at the top and bottom surfaces of the laminate and shear correction factors are no longer required. A general straight-sided quadrilateral domain is mapped into a square domain in the computational space using a four-node master plate, employing a geometric transformation. The displacement field components are approximated by sets of beam characteristic orthogonal polynomials generated using the Gram–Schmidt procedure. The use of Ritz method allows a high spectral accuracy and faster convergence rates than local methods such as finite element. The algorithm developed is quite general, free of shear locking and can be used to obtain natural frequencies and modal shapes of laminated plates having various material parameters, geometrical planforms, length-to-thickness ratios and any combinations of free, simply supported and clamped edge support conditions.

Through several numerical examples, the capability, efficiency and accuracy of the formulation are demonstrated. Convergence studies and comparison with other existing solutions in the literature suggest that the present algorithm is robust and computationally efficient.

1. Introduction

Due to high strength and stiffness-to-weight ratio of the fiber reinforced composite materials, laminated plates of various shapes composed of these materials have found wide applications as structural members in different engineering fields. Therefore a thorough understanding of the vibration behaviors of composite laminated plates with various geometrical planforms, boundary conditions and length-to-thickness ratios is of great interest for the designers to realize proper and comparatively accurate design of machines and structures. One of the important features of fiber reinforced composite laminates is that they are weak in shear i.e. the value of shear modulus is sufficiently low compared to that of extensional rigidity. Because of this reason the transverse shear deformations are much pronounced for laminated plates than for isotropic ones, so this effect becomes quite significant and it should be considered in the analysis in a proper manner.

The first order shear deformation theories (FSDT) [1,2] which assumes constant transverse shear deformation through the entire thickness of the laminate, violates stress free boundary conditions at the top and bottom surfaces of the plate, and hence shear correction factors need to be involved. Therefore, the accuracy of the FSDT directly depends on this factor which varies with the loading conditions, laminations sequences and boundary conditions as informed by Pai [3]. In order to consider the non-linear transverse shear stress distribution accurately and hence eliminate the requirement of shear correction factor, various higher order shear deformation theories (HSDT) have been proposed. In this kind of plate theories, the transverse shear strain is made non-uniform by taking non-linear through thickness variation of the in-plane displacements. In general, these displacements are approximated by polynomial [4–10] and non-polynomial expressions [11–21]. A clear and precise description of the models, types and classes of theories can be found in the articles by Carrera [22], Reddy and Arciniega [23], and Wanji and Zhen [24].

Along with the development of plate theories, there has been significant development towards the solution methodologies. Particularly, various researchers have addressed the problem of vibration analysis of laminated plates employing HSDT. Liu et al. [25] employed a mesh-free radial basis function method to analyze plates using the third-order shear deformation plate theory. Kant and Swaminathan [26] presented analytical solutions for free vibration of simply supported laminated composite and sandwich plates based on a higher order theory. A Navier type and finite element solutions were proposed by Grover et al. [20] to obtain the free vibration response of laminates, employing an inverse

* Corresponding author. Fax: +54 387 4255351.
E-mail address: ritarango@conicet.gov.ar (R.F. Rango).

http://dx.doi.org/10.1016/j.ijmecsci.2015.09.018
0020-7403/

hyperbolic shear deformation theory. Xiang and Wang [27] employed the trigonometric shear deformation theory (TSDT) and the inverse multiquadric radial basis function (RBF) to predict the free vibration behavior of symmetric laminated plates. RBF have been also applied by Ferreira et al. [28,29] with higher order shear deformation theories for the analysis of laminated composite beams and plates. Thai et al. [21] used an isogeometric analysis for studying laminated composite and sandwich plates within the framework of a new inverse trigonometric theory proposed by the authors. Based on the third order shear deformation theory of Reddy [30], Dai et al. [31] used the moving least-squares method to construct the shape functions for analyzing the static and free vibration behavior of laminated plates.

Carrera [32] developed a powerful formal technique to handle, in a unified manner, an infinite number of equivalent single layer and layer wise axiomatic plate and shell theories with variable-kinematic properties. This technique was employed in several research articles; in particular Dozio and Carrera [33] presented a variable-kinematic Ritz formulation based on two-dimensional higher-order layerwise and equivalent single-layer theories to accurately predict free vibration of skew multi-layered plates.

Recently, an energy-oriented modified Fourier method has been developed to solve vibration problems of generally laminated plates and shells with arbitrary boundary conditions [34–41].

Although there are many references available on free vibration analysis of laminated composite plates, most of them deal with rectangular cross-ply composite plates; while references about the dynamic behavior of general thick quadrilateral laminated composite plates, with any combination of boundary conditions, are rather limited.

Nallim et al. [42] and Nallim and Oller [43] developed a general algorithm for the study the behavior of thin laminated plates based on the Ritz method and a mapping technique to encompass different quadrilateral plate domains. It is important to emphasize that the Ritz method provides upper-bound vibration frequencies [44]. Furthermore, as stated by Dozio and Carrera [33], relying on a global approximation, Ritz method has a high spectral accuracy and converges faster than local methods such as finite element, and it can be quite suitable to provide benchmark references values. For these reasons in this paper a general variational Ritz approach, with sets of beam characteristic Gram–Schmidt orthogonal polynomials as approximating functions, for the dynamical analysis of thick quadrilateral laminated plates is presented. The analysis is based on the trigonometric shear deformation theory [45,46] and the natural coordinates to define a single domain comprehending laminates with different geometries. This variational approach allows investigating the free vibration characteristics of several composite laminated plates of different geometrical planforms, with any combination of boundary conditions and length-to-thickness ratios. The algorithm developed is also very efficient from a computational point of view since a sparse eigenvalue-problem is obtained. Besides, this variational approach can be quite suitable during preliminary design studies and parametric analysis. To demonstrate the validity and efficiency of the proposed formulation, several numerical examples are solved and some of them are verified with results from others authors. New solutions are also presented for future comparison purpose.

2. Formulation

2.1. Description of the model

A quadrilateral laminated plate of total thickness h and arbitrary boundary conditions is considered. The plate has, in general, different side lengths as shown in Fig. 1(a). The laminate consists of N_l layers, which are assumed to be made of orthotropic material, where the fiber angle of the kth layer counted from de surface $z=-h/2$ is β measured from the x axis to the fiber orientation, with all laminae having equal thicknesses, and symmetric lamination of plies is considered. The present study is based on the trigonometric shear deformation theory [45,46]. In this theory the displacement field of a symmetric laminated composite plate can be defined as

$$\begin{aligned}
\overline{u}(x,y,z,t) &= u_0 - z\frac{\partial w_0(x,y,t)}{\partial x} + \sin\frac{\pi z}{h}\overline{\phi}_x(x,y,t)\\
\overline{v}(x,y,z,t) &= v_0 - z\frac{\partial w_0(x,y,t)}{\partial y} + \sin\frac{\pi z}{h}\overline{\phi}_y(x,y,t)\\
\overline{w}(x,y,z,t) &= w_0(x,y,t)
\end{aligned} \tag{1}$$

where $\overline{u}$ and $\overline{v}$ are the in-plane displacements at any point (x,y,z), u_0 and v_0 denote the in-plane displacement of the point $(x,y,0)$ on the mid-plane, $\overline{w}=w_0$ is the transverse deflection, $\overline{\phi}_x$ and $\overline{\phi}_y$ are the rotations of the normals to the midplane about the y and x axes, respectively. This paper only considers symmetric laminates, therefore we can disregard contributions of u_0, v_0. During free vibration, the displacements are assumed split in the spatial and temporal parts, being them periodic in time:

$$\begin{aligned}
w_0(x,y,t) &= w(x,y)\sin(\omega t)\\
\overline{\phi}_x(x,y,t) &= \phi_x(x,y)\sin(\omega t)\\
\overline{\phi}_y(x,y,t) &= \phi_y(x,y)\sin(\omega t)
\end{aligned} \tag{2}$$

where ω is the radian natural frequency.

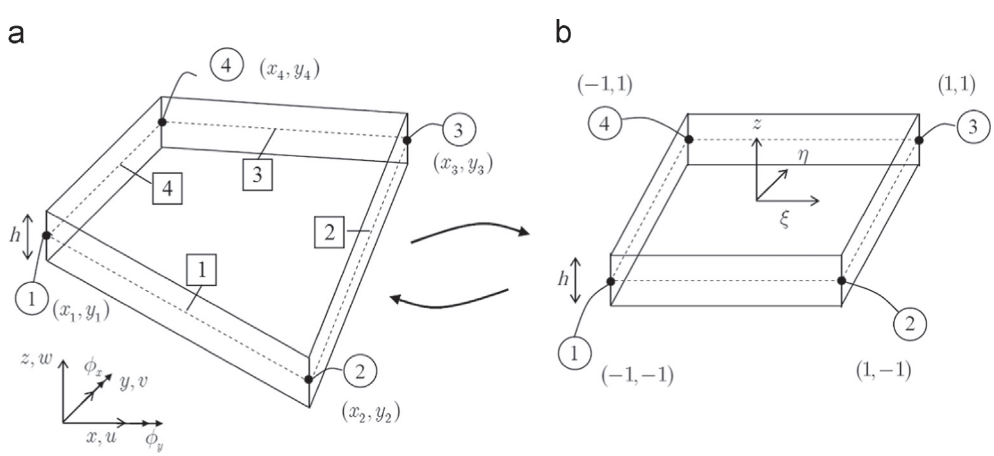

Fig. 1. General quadrilateral thick laminated plate element in: (a) Cartesian coordinates (x,y), and (b) natural coordinates (ξ,η).