International Journal of Mechanical Sciences 104 (2015) 138–146

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Mode jumping analysis of thin film secondary wrinkling

C.G. Wang [a,c,*], H.F. Tan [a,c], L. Lan [a], L. Li [b]

[a] Center for Composite Materials, Harbin Institute of Technology, Harbin 150001, China
[b] Department of Materials Science and Engineering (MSE), Cornell University, Ithaca, NY 14850, USA
[c] National Key Laboratory of Science and Technology on Advanced Composites in Special Environments, Harbin Institute of Technology, Harbin 150080, China

ARTICLE INFO

Article history:
Received 10 January 2015
Received in revised form
9 May 2015
Accepted 14 October 2015
Available online 20 October 2015

Keywords:
Thin film
Secondary wrinkling
Mode jumping
Bifurcation theory

ABSTRACT

In this paper, a mathematic expression is presented to describe the mode jumping and the equilibrium path reversal characteristics of thin film secondary wrinkling based on the non-linear bifurcation buckling theory. With the usage of variable transformation and the introduction of dimensionless parameters, the non-linear Von-Karman equilibrium equation considering the two-direction loading characteristics, is transformed into a non-linear boundary value problem with zero trivial solution. Based on the bifurcation theory, the eigenvalue problem is addressed through the linearization of non-linear boundary value problem. An approach to critical load prediction of thin film secondary wrinkling is proposed by introducing a critical aspect ratio parameter, since the mechanism of thin film secondary wrinkling is double eigenvalues splitting. Furthermore, this paper presents an analysis in detail on the critical load of the rectangular thin film secondary wrinkling under shear, indicating that the critical load is linearly proportional to the initial stretching displacement, while its relationship with the size of the free edge is nonlinear. The validity of this critical load prediction approach is verified via the digital image correlation experiment. The results provide strong supports for the controlling and tuning of the wrinkles for high precision pre-tensioned thin film structures.

1. Introduction

Currently, thin film is widely used for various applications, such as large deployable antennas, solar arrays and telescope lenses [1,2]. However, with the existence of small compressive stress, the wrinkles (or the surface undulations) are easy to develop in thin film structures owing to negligible bending stiffness. According to the experimental observations and simulations [3–7], the wrinkling mode jumping exists in the post-wrinkling stage, which results in a complicated post-wrinkling behavior of thin film structures. The performance and reliability of high precision flexible thin film structures could be affected by two main factors: the wrinkling evolution and the characteristics of wrinkling wave pattern [8–11]. To better control and tune the wrinkles on the thin film surface as well as explore the evolution of thin film wrinkling, the research on characteristics and mechanism of thin film wrinkling mode jumping is of great significance.

The wrinkling is a local buckling instable for thin films (small but finite bending stiffness) rather than membranes (no bending stiffness). It is assumed to be a bifurcation of plane stress deformation to an out-of-plane bending-type deformation. The bifurcation buckling theory coupled with the thin plate/shell model is able to obtain the evolution characteristics of the post-wrinkling behavior and the detailed information of wrinkling wave pattern [3,5,12–14]. As a phenomenon of local bifurcation buckling, the evolution progress of primary wrinkling is multi-faceted in the post-wrinkling stage. It has been observed by numerous researchers that the patterns of primary wrinkling can undergo sudden changes, namely secondary wrinkling [4–7]. Characteristics of thin film secondary wrinkling are similar to thin plate secondary buckling [15–18], such as the mode jumping in the post-buckling stage. However, there are still some differences between thin film secondary wrinkling and thin plate secondary buckling. For example, thin film secondary wrinkling can lead to the local mode jumping and it has a complicated evolution process.

Two methods have been applied to investigate the secondary wrinkling behavior of thin plate. One is the Galerkin method combined with the energy principle, and the other is the bifurcation method. Based on the energy principle, the critical point of secondary buckling for the thin plate can be obtained when the total potential energy in the post-buckling stage is zero. It is theoretically proved that the secondary buckling of thin plate is a snap-type buckling phenomenon accompanied by an abrupt change of wave-form due to the instability in the post-primary-buckling range [19–25]. Based on the bifurcation method, the

* Corresponding author.
E-mail address: wangcg@hit.edu.cn (C.G. Wang).

http://dx.doi.org/10.1016/j.ijmecsci.2015.10.007
0020-7403/

secondary buckling of thin plate is closely associated to a multiple primary branching point. A certain class of secondary buckling points is generated by a splitting process of a multiple-primary buckling point. Jumping in wave-form after the primary buckling can be explained with these secondary branches [26–32]. These two methods are good references to predict the secondary wrinkling behaviors of the pre-stressed thin film. Recently, study on the secondary wrinkling behaviors of pre-stressed thin film is reported about its phenomenon interpretation, which is a field lacking of deep theoretical research [3–7]. In our recent work, a numerical judgment method of the secondary wrinkling of pre-stressed thin film based on the numerical and experimental studies was proposed [6]. However, the essential mechanism of the secondary wrinkling of pre-stressed thin film still remains unclear.

In this paper, a mathematic description of the characteristics of thin film secondary wrinkling is established based on the nonlinear bifurcation buckling theory. The mechanism of thin film secondary wrinkling is then elaborated using the bifurcation theory via the analysis of an eigenvalue problem derived by the nonlinear Von-Karman equilibrium equations. Finally, a new critical load prediction method of thin film secondary wrinkling will be proposed based on this mechanism. The specific critical load of the secondary wrinkling of the rectangular thin film shear wrinkling will be provided, followed by the verification of its reasonability and availability via the digital image correlation experiment.

2. Thin film secondary wrinkling phenomenon and its mathematical description

2.1. Wrinkling wave mode jumping

According to the numerical simulation method proposed by Wang et al. [5], the evolution progress of thin film secondary wrinkling can be obtained with the rectangular thin film shear wrinkling taken into account. The size and material parameters are shown in Fig. 1. The bottom and upper edges are fixed. Thin film wrinkling simulation is typically composed of three steps. Firstly, an initial stretching displacement v_1 is applied on the upper edge to provide the pre-stress for the thin film structures. Next, certain small disturbing forces are applied on the surface of thin film. The deflection of thin film induced by these disturbing forces should be approximate to or less than the thickness of thin film. Finally, the upper edge of thin film is subjected to a horizontal displacement load u_1. In order to overcome the simulation non-convergence, the displacement load u_1 is applied step by step. In this analytical case, the initial stretching displacement v_1 is 0.15 mm, and the displacement load u_1 is equal to 0.1 mm for each step.

The evolution of thin film secondary wrinkling obtained with numerical simulation is shown in Fig. 2. A sudden change phenomenon of wrinkling mode occurs in the post-wrinkling stage, which is called the wrinkling wave mode jumping. As shown in Fig. 2, the wrinkling wave mode jumping can be observed in the local region of primary wrinkling.

The transverse section I-I (Fig. 1) in the wrinkling wave mode jumping region is selected. The wrinkling wave mode of this selected transverse section is shown in Fig. 3a. It is demonstrated that the primary wrinkling wave pattern is symmetric (both before and after the mode jumping) within the domain of the plot (roughly $140 < X < 190$). This appears to be a simple change of the buckling mode shape as the wave number increases by a half wave. Arbitrary four nodes are selected on the thin film surface (shown in Fig. 2a). Nodes B and C are in the wrinkling wave mode jumping region. Fig.3b indicates the post-wrinkling equilibrium paths of these four nodes. It is illustrated that equilibrium paths of nodes B and C undergo abrupt snap-through progress in the post-wrinkling stage which results in the reversal jumping of equilibrium path.

2.2. Mathematical description of thin film secondary wrinkling

In the post-wrinkling process, the phenomenon of sudden wrinkling wave mode jumping on the thin film surface is defined as the secondary wrinkling. According to the characteristics of thin film secondary wrinkling obtained in Section 2.1, a mathematical description is proposed as follows.

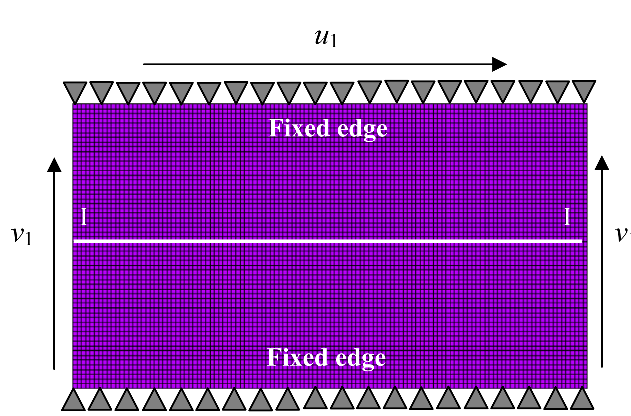
$$\Delta w_i^{n_i} = w_i^{n_i} - w_{i-1}^{n_i} \tag{1a}$$

$$w_i^{n_i} \cdot w_{i+1}^{n_i} < 0 \tag{1b}$$

$$F_{cr-II}^{n_i} = F_i^{n_i}\{\max|\Delta w_i^{n_i}|\}\big|_{i=1}^{i=n} \tag{1c}$$

where i is the iteration step, n is the total number of iteration step, $w_i^{n_i}$ and $F_i^{n_i}$ are the displacement component in the normal direction and the load for arbitrary iteration step i and arbitrary node n_i.

For the node in the wrinkling wave mode jumping region, the equilibrium path reversal characteristics and the snap-through characteristics are reflected by Eqs. (1b) and (1c), respectively. The region containing these nodes whose equilibrium paths satisfy Eq. (1b) is defined as the secondary wrinkling region; meanwhile if the equilibrium paths of the nodes also satisfy Eq. (1c), the load



E(GPa)	3.53
v	0.3
h(μm)	50
width(mm)	150
length(mm)	300

Fig. 1. Size and material parameters of the simulation analytical mode.