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In-plane nonlinear localised lateral buckling under thermal loading of rail tracks modelled as a sandwich column

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ABSTRACT

The lateral buckling problem for continuous welded rail tracks under thermal loading has been well researched and is known to involve a localisation buckling phenomenon where only a limited region of the track buckles. In this paper, a sandwich column model is formulated to model the rail track-tie structure as sandwich column. The constitutive relations for the thermally induced stresses and finite strain are based on a hyperelastic constitutive model. A hyperbolic function for the nonlinearity of the axial and lateral resistance between the tracks and ballast was used. A critical track length is proposed beyond which the localisation is unaffected. Increasing the track length or changing the boundary conditions does not influence the localised buckling behaviour for track lengths beyond the critical length. The use of fasteners with large rotational stiffness substantially increases the lateral stability of the track-tie structure. Nonlinear numerical solutions are compared with the results in the literature. The variation in the axial compressive force within the localisation zone in the rail is studied and discussed. Parametric studies are performed on examples to identify the influence of track imperfections on the localised lateral buckling and safe temperature change.

1. Introduction

With the rapid improvement of technology in the rail industry, many of the conventionally jointed tracks that once required bolted connections have been replaced by continuous welded rail tracks. The continuous welded rail track system, which has become the best choice for high-speed rail, has many advantages when compared with jointed tracks; it decreases maintenance costs, increases the service life of the track and vehicle component, reduces the energy consumption and improves the passengers' comfort. However, a noteworthy drawback of this kind of track is its tendency to buckle if the environmental temperature varies significantly. When the temperature increase is sufficiently high, lateral buckling instability in the horizontal plane may occur [1].

It is imperative to model accurately the nonlinear localised buckling of rail tracks under thermal loading as this leads to efficient design. Kerr [1] proposed an analytical method to investigate the buckling of tracks using a beam model. Kerr assumed that the track only buckles in the lateral direction in the buckled region in which the axial compressive load is assumed to be constant, whereas the track in the adjoining regions only deforms axially. In addition, the axial resistance was neglected in the buckled region. Using these assumptions, Kerr presented a basic theory and proposed a useful methodology to study the lateral buckling of tracks, adopted by many researchers. Kerr further developed his formulation using a beam track model and buckled region assumptions for thermal buckling of tracks to cover a number of practical cases [2–4].

A limitation of the beam model is that the effects of sleepers and fasteners between the rails on the nonlinear buckling deformation is ignored. In the beam model, the lateral bending rigidity of the track was taken as twice the lateral rigidity of one rail. However, for the real case, the rail-tie structure contributes to the stiffness as the fasteners connecting both rails significantly increases the shear stiffness of the equivalent frame-type rail structure [5–7]. Kerr also developed the basic formulations for the frame-type rail structure and crosstie tracks [5,8,9]. Grissom and Kerr [10] applied the frame-type equations to investigate the antisymmetric buckling of rail tracks using the buckled and adjoining region assumptions. In the frame-type model, the axial force in each rail was taken as the same before buckling. However, the axial forces in both rails change in different ways if the tracks buckle. In addition, it was found that the rotational fastener resistance significantly influences the nonlinear localised buckling. Kish and Samavedam et al. [11–14] carried out a number of experiments to investigate buckling of rail tracks and also developed an analytical formulation based on Kerr's assumptions. It was found that straight tracks typically buckle in a symmetric mode 3

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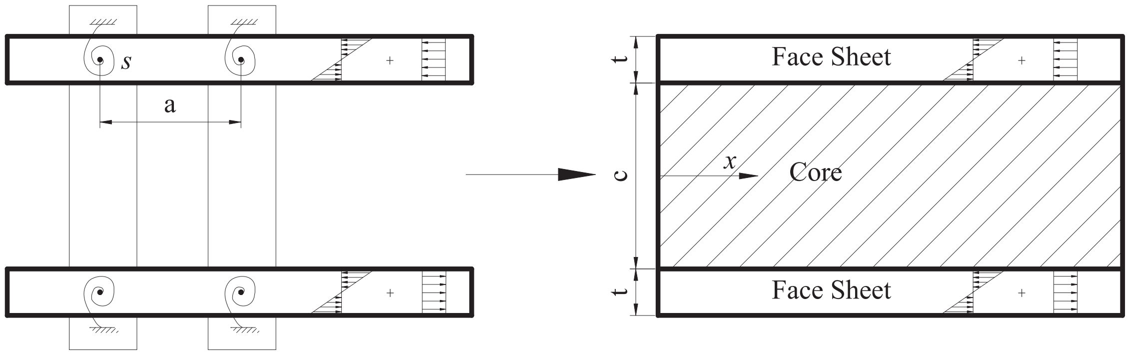

Fig. 1. Sandwich model for the rail-tie track system.

(see Kerr [1]), whereas curved tracks buckle in symmetric mode 1 (see Kerr [1]). The experiments can predict the tendency of the deformation but cannot offer a accurate prediction for the in-service rail tracks, as the real tracks are much longer than the samples in the tests. The finite element method has been adopted by many researchers to investigate buckling of rail tracks [15–21]. Finite element modelling is capable of including the stiffness of the fasteners, the lateral and axial resistance and track irregularities. This method requires complex and time consuming modelling.

To get results which better model the nonlinear localised buckling behaviour of rail tracks and the interaction with fasteners, a sandwich beam model is developed in this paper. In the rail-tie track system as shown in Fig. 1 the fasteners generate a rotational resistance in addition to exerting the resistance in the axial and lateral directions. Thus, the bending moment is carried not only by the bending stresses of each rail, but also by the axial forces in the rails. The rotational stiffness of the fasteners should be taken into account in the analysis. The rail-tie system is similar to a build-up column that behaves like a sandwich column [22]. As illustrated in Fig. 1 the face sheets in the sandwich model are equivalent to the rails and the geometry and the material properties of face sheets can be calculated from the rails directly. The rotational stiffness of the fasteners is modelled by an equivalent shear modulus of the sandwich core. Employing the method presented by Bazant [6, 23], the equivalent shear modulus of the core is as follows:

$$G_c A_c = \frac{2s}{a} \tag{1}$$

where G_c and A_c are the shear modulus and the cross-sectional area of the core, respectively, s is the rotational stiffness of the fasteners and a is the spacing between fasteners. The ties are assumed not to carry any axial force. Therefore, the cross-sectional area and young's modulus of the core are set as zero.

In this paper, a sandwich model is developed and using a new approach in which the Kerr buckled and adjoining region assumptions are abandoned and no assumption is made for the axial compressive force or the displacements. Both symmetric and antisymmetric localised buckling are investigated. A critical track length is proposed for the localised lateral buckling analysis. It is shown that localised buckling can be induced on the long rail tracks subjected to thermal loading and that there is a critical track length. The influence of the rotational fastener stiffness on the localised buckling behaviour is also investigated through a parametric study. The results obtained in this paper are compared with the results in the literature. The results for the axial compressive force distribution in each rail during the formation of a localised lateral deformation along the track is examined and discussed. The significant effect of imperfections on the symmetric and antisymmetric buckling behaviour and safe temperature change are investigated.

2. Sandwich model for rail tracks

In order to investigate the nonlinear buckling of rail tracks, an equivalent sandwich model including thermal effects is firstly established. A strain energy density for isotropic hyperelastic materials under finite strain was proposed by Attard [24–28] and used to derive constitutive relationships for the buckling problem of the column and arch. The hyperelastic formulations were further extended to investigate the buckling of sandwich columns [29]. In this paper, the hyperelastic formulation for a sandwich column by Attard is used to model the rail track-tie structure and extended to include thermal loading.

2.1. Two dimensional hyperelastic mechanics

The kinematics for beams with thermal loading effect were developed in the reference [30]. Here the hyperelastic mechanics in the reference [30] is summarised. Consider a two-dimensional plane continuum with an initial Cartesian coordinate system. The relationships between the stretches and bending angle θ and shear angle φ (clockwise) with the displacement gradients are:

$$\lambda_1 \sin(\varphi-\theta) = u_{2,x} \quad \lambda_1 \cos(\varphi-\theta) = 1+u_{1,x} \quad \lambda_2 \sin\theta = u_{1,y} \quad \lambda_2 \cos\theta = 1+u_{2,y} \tag{2}$$

where

$$\lambda_1 = \sqrt{(1+u_{1,x})^2+u_{2,x}^2} \quad \lambda_2 = \sqrt{(1+u_{2,y})^2+u_{1,y}^2} \quad \tan(\varphi-\theta) = \frac{u_{2,x}}{1+u_{1,x}} \quad \tan\theta = \frac{u_{1,y}}{1+u_{2,y}} \tag{3}$$

in which u_1 and u_2 are displacement components in the x and y directions and λ_1 and λ_2 are the stretches in the x and y directions, respectively. The subscript comma as in $u_{1,x}$ indicates differentiation with respect to the proceeding variable, here x.

When the thermal load is included, the deformation can be divided into the mechanical part and the thermal part. The mechanical part of deformation tensor $\mathbf{F}^{e*}$ is written as:

$$\mathbf{F}^{e*} = \begin{bmatrix} \lambda_{1T}\cos\varphi & 0 \\ \lambda_{1T}\cos\varphi & \lambda_{2T} \end{bmatrix} \tag{4}$$

where:

$$\lambda_{1T} = \frac{\lambda_1}{1+\alpha_o \Delta T} \quad \lambda_{2T} = \frac{\lambda_2}{1+\alpha_o \Delta T} \tag{5}$$

The strain energy density function for a compressible isotropic neo-Hookean material [24,25,31] is employed. Substituting the mechanical part of the deformation tensor into the strain energy