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Isogeometric finite element method for buckling analysis of generally laminated composite beams with different boundary conditions

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ABSTRACT

In this paper, isogeometric finite element method (IGA) based on Non-Uniform Rational B-splines (NURBS) basis function is applied for the buckling analysis of generally laminated composite beam with various boundary conditions. A beam element with four degrees of freedom per control point is investigated, which has considered the bending-torsion deformation. The model for the buckling analysis of laminated composite beam is detailed by the principle of virtual work. Several numerical examples of symmetric and anti-symmetric, cross-ply and angle-ply composite beam are performed. Numerical results of critical buckling loads and mode shapes are presented, and compared with other available results to show the efficiency and accuracy of the present IGA approach. In addition, the impacts of the modulus ratios, slenderness ratios, stacking sequence and the fiber angle, especially the Poisson effect on the critical buckling loads of composite beam are clearly demonstrated. It should be noted the results with the Poisson effect neglected are only suit for the cross-ply composite beams. And the benchmark solutions presented in this work can be used as a reference for the buckling analysis of laminated composite beams in future.

1. Introduction

The fiber-reinforced composite structures formed by combining two or more materials have been widely used in applications ranging from aerospace structural frames to mechanical, marine and automobile structural components due to their high strength, high stiffness to weight ratio, excellent durability and other outstanding engineering properties obtained by the proper selection of the fiber orientation and stacking sequence of the layers [1]. Without doubt, the potential benefits and practical importance of composite structures require a deep understanding of the vibration characteristics and buckling responses of laminated composite beams, especially for those subjected to dynamic loads in the complex environment such as the aircraft wing and the vehicle axles. This type of analysis is also an important part of the reliable optimization design of such structures.

Various theoretical models have been developed by many investigators for the free vibration and buckling analysis of the fiber-reinforced laminated composite plate structure, e.g. see [2–7]. Some other investigators explored the mechanical behaviors of the fiber-reinforced laminated composite beam. First, J.N. Reddy and A.A. Khdeir developed an exact analytical solutions based on the refined beam theories to study the free vibration [8], buckling [9] behavior of cross-ply laminated composite beams with arbitrary boundary conditions. Numerical results are presents for references. Abramovich et al. [10] introduced the exact element method based on the first-order shear deformation theory for the vibration and buckling analysis of cross-ply non-symmetric laminated composite beams. The influence of boundary conditions, materials and layup sequence on the buckling loads and natural frequencies is also considered. Karama et al. [11,12] proposed a new transverse shear stress continuity model to study the beading, buckling and free vibration analysis of laminated composite beams. Results from the analysis show that the proposed model is more precise than finite element method. Matsunaga put forward the global higher order approximate theories to predict the natural frequencies, buckling stresses of laminated composite beam [13] and composite plate [14,15]. Jun [16,17] used different shear deformation theories to predict the natural frequencies of laminated composite beams. And good results have been obtained in their work when compared with other available results in literatures. Metin Aydogdu applied the Ritz method to study the free vibration [18], buckling [19] and thermal buckling analysis [20] of cross-ply laminated composite beams with general boundary conditions. The numerical results show that the Ritz method can be used to solve those problems with enough accuracy. Zhen and

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Wanji [21] obtained analytical solutions to the natural frequencies and buckling analysis of laminated composite beam and sandwich beams based on the global local higher order theory. Numerical results showed that the global higher order theory can achieve higher accuracy on the free vibration and buckling analysis than other displacement-based beam theories. Thuc Vo and Thai [23,24,36] introduced the refine shear deformation theory for the static, free vibration and buckling of composite beams. Numerical results were obtained to investigate the effect of the fiber orientation and modulus ratio on the natural frequencies, critical buckling loads.

More recently, Hughes et al. [25,26] introduced a new technique called isogemetric analysis (IGA) into the engineering and science disciplines, which has attracted many researchers. The IGA method first combined the NURBS basis functions most commonly used in the CAGD and traditional finite element method. As a result, various advantages can be obtained, e.g., better handling of complicated geometry areas, can provide more robust and accurate results, effective refinement algorithms without any interaction with the CAD system. Based on that, the IGA method has been widely used in many applications such as structural vibration [27,28], plate and shell theory [29–31], structural optimization [32] and so on.

Most of the previous works are focused on the free vibration analysis of composite beams or on some special cases such as cross-ply beams, neglecting the Poisson effect. To the authors' knowledge, there are no or few studies focused on the buckling analysis of angle-ply laminated composite beams in the literature. This work is partly motivated by Refs. [16,33]. In their work, they studied the free vibration of generally laminated composite beams based on a first order shear deformation theory. Numerical results of the natural frequencies had been shown to be in good agreement with the data available in the literature. In this work, we introduce isogeometric method for the buckling analysis of laminated composite beams with different boundary conditions, where the same NURBS basis function is applied for the approximation of both the field variables and the geometry. With NURBS basis functions, a C^1 continuity element with three control points and four degrees of freedom per control point is constructed. The validity and accuracy of the present IGA method for buckling analysis of laminated composite beam is testified by same carefully selected examples. From numerical results, the present element can predict very accurate critical buckling loads values and prove to be suit for both thick and thin laminated composite beams. In addition, the influences of various slenderness ratios, modulus ratios, stacking sequence and the fiber angle, especially the Poisson effect on the critical buckling loads of laminated composite beam are also considered. It should be emphasized that, unlike other numerical approach, the present isogeometric formulation do not need any complicated derivation, which is easy to be accepted by engineering researchers.

2. Mathematical formulation for laminated composite beam

In this work, a laminated composite beam with rectangular cross-section of width b and thickness h is considered. As shown in Fig. 1, the beam is referred to a system of right-handed Cartesian coordinate originating on the mid-plane of the beam, with its x-axis being coincident with the beam axis.

The present formulation is based on the first-order shear deformation theory, where the assumed displacement fields are presented as follows

$$u(x,z,t) = u_0(x,t) + z\gamma_x(x,t)$$
$$v(x,z,t) = z\psi(x,t)$$
$$w(x,z,t) = w_0(x,t) \quad (1)$$

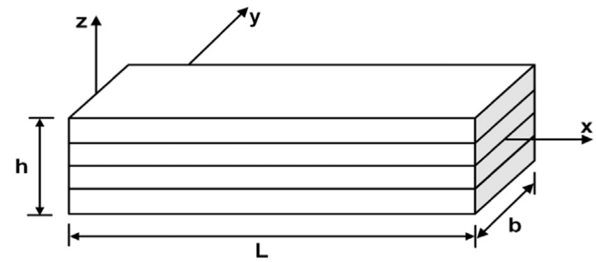

Fig. 1. Geometry of a laminated composite beam.

where t is the time, u_0 is the mid-plane longitudinal displacement, w_0 is the deflection of the beam in the z direction and γ_x and ψ are the rotation of the normal to the mid-plane in the x and y direction.

For the first-order shear deformation beam, there are only three nonzero strains are given by

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x, \gamma_{xz} = \partial w_0/\partial x + \gamma_x, \gamma_{xy} = z\kappa_{xy}$$
$$\varepsilon_x^0 = \partial u_0/\partial x, \kappa_x = \partial \gamma_x/\partial x, \kappa_{xy} = \partial \psi/\partial x \quad (2)$$

For this beam theory, due to neglect the in-plane forces N_y, N_{xy} and bending moments M_y, the constitutive equation can be written as

$$\begin{Bmatrix} N_x \\ M_x \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{B}_{11} & \overline{B}_{16} \\ \overline{B}_{11} & \overline{D}_{11} & \overline{D}_{16} \\ \overline{B}_{16} & \overline{D}_{16} & \overline{D}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \kappa_x \\ \kappa_{xy} \end{Bmatrix} \quad (3)$$

$$Q_{xz} = kA_{55}\gamma_{xz} = kA_{55}(\partial w_0/\partial x + \gamma_x) \quad (4)$$

where N_x, M_x, M_{xy} are the normal force, bending moment and twisting moment, respectively. Q_{xz} is the transverse shear force per unit length. And $k = 5/6$ is the shear correction factor, which is introduced for correction of the transverse shear stiffness. $\overline{A}_{11}, \overline{B}_{11}$, $\overline{D}_{11}$ represent the transformed extensional, flexural-extensional coupling, flexural stiffness coefficients of the laminated composite beam, respectively. And they are the coefficients of the matrix $[\boldsymbol{A} - \boldsymbol{B}\boldsymbol{C}^{-1}\boldsymbol{B}^T]$. The matrices $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$ are given as follows

$$\boldsymbol{A} = \begin{bmatrix} A_{11} & B_{11} & B_{16} \\ B_{11} & D_{11} & D_{16} \\ B_{16} & D_{16} & D_{66} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} A_{12} & A_{16} & B_{12} \\ B_{12} & B_{16} & D_{12} \\ B_{26} & B_{66} & D_{66} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} A_{22} & A_{26} & B_{22} \\ A_{26} & A_{66} & B_{26} \\ B_{22} & B_{26} & D_{22} \end{bmatrix}$$

The laminated composite beam stiffness coefficient $A_{ij}, B_{ij}, D_{ij}, (i,j = 1,2,6)$ and transverse shear stiffness coefficient A_{55} are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij}(1, z, z^2)dz, \quad i,j = 1,2,6,$$
$$A_{55} = \int_{-h/2}^{h/2} \overline{Q}_{55} dz \quad (5)$$

The transformed reduce stiffness coefficients of the layers $\overline{Q}_{ij}$ are denoted by [34]

$$[\overline{Q}] = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} = [T]^{-1} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} [T]$$
$$\overline{Q}_{55} = G_{13}\cos^2\theta + G_{23}\sin^2\theta \quad (6)$$

where $[T]$ is the transformation matrix given as follows:

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

in which $[T]^{-1}$ is the inverse of the matrix $[T]$, $m = \cos\theta$ and $n = \sin\theta$, θ is the angle between the fiber direction and longitudinal axis of the beam, and the reduced stiffness coefficients Q_{ij}