International Journal of Mechanical Sciences 104 (2015) 200–206

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Thermoelastic response of thin plate with variable material properties under transient thermal shock

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ARTICLE INFO

Article history:
Received 25 May 2015
Received in revised form
12 October 2015
Accepted 21 October 2015
Available online 28 October 2015

Keywords:
Generalized thermoelasticity
Variable material properties
Transient thermal shock
Temperature dependency
Thin plate

ABSTRACT

This paper is to investigate the thermoelastic response of an elastic medium with variable material properties under the transient thermal shock. The governing equations involving temperature-dependent properties are proposed by the Clausius inequality and generalized theory of thermoelasticity with one thermal relaxation time, where the higher order expansion with respect to increment temperature of the Helmholtz free energy is used to describe the relations of each material parameter with real temperature. The problem of a thin plate composed of titanium alloy, subjected to a sudden temperature rise at the boundary, is solved. The propagations of thermoelastic wave and thermal wave, as well as the distributions of displacement, temperature and stresses, are obtained and discussed. The comparison of present results with those obtained from the case of constant material properties are conducted to reveal the effect of the temperature dependency of material properties on thermoelastic response. The comparison with the results obtained from the case that each material parameter is the function of fixed temperature is also conducted to explain the coupling effect between variable material properties and temperature distribution.

1. Introduction

The requirement for thermo-mechanical properties of materials is growing with its extensive applications in aerospace, nuclear reactors, pressure vessels and pipes, and other engineering areas. In recent years, the research on thermoelastic behavior of materials in some severe environment, such as elevated temperature, severe thermal gradient and magnetic field, has drawn great attention [1–3]. Experiments [4] have proven that thermal signal propagates in an elastic medium with a finite speed when heat conduction takes place in a short time interval or the high heat flux. Meanwhile, the material properties such as the modulus of elasticity, Poisson's ratio, the thermal conductivity and the specific heat are no longer constants at same conditions [5]. The Non-Fourier effect [6] induced by finite heat propagation, as well as the temperature dependency of material properties, would have significant influence on thermoelastic response [7–9]. Based on the generalized theories of thermoelasticity [10–12], admitting the finite propagation velocity of thermal signal, Ezzat et al. [7], Youssef [13], Aouadi [14], Othman and Kumar [15], Allam et al. [16] and Abbas [17,18] investigated different thermoelastic problems involving variable material properties, respectively, where the assumption that material parameters are the linear functions of reference temperature were used to simplify the solutions of governing equations with variable material properties. Xiong and Tian [19], and He et al. [20] considered the linear relations of material parameters with real temperature, and pointed out that the effect of temperature dependency of material properties on thermoelastic response would be enhanced. Sherief and El-Latief [21], Zenkour and Abbas [22] also solved a generalized thermoelastic problem with variable material properties, where the linear function and the exponential function of variable thermal material parameters with real temperature were used to reveal the influence of temperature dependency on thermoelastic behavior.

It is noted that the governing equations, used to deal with variable material properties in these investigations [7,13–22], are derived by introducing some specific functions, which describe the relations of each material parameter with reference temperature or real temperature, into the formulations with constant material properties. This treatment neglects the influence of variable material properties on the inherent form of governing equations. Based on the Clausius inequality and conventional theory of thermoelasticity [23], Dillon [24] and Wang [25] derived the governing equations of an isotropic medium with variable material properties, respectively, where the higher order expansions of the

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http://dx.doi.org/10.1016/j.ijmecsci.2015.10.013
0020-7403/

Helmholtz free energy with increment temperature were introduced to obtain the functions of each parameter with real temperature. They pointed out that the forms of constitutive equation and associated thermoelastic equation were not changed, although they were nonlinear under variable material properties, however, the new term would generate in the right side of temperature equation when involving variable material properties. This new term can be regarded as an additional heat source and would have some effect on thermoelastic response, especially for the large deformation rate. Therefore, it is necessary to consider the changes of governing equations in the following investigations of thermoelastic problem with variable material properties.

In this paper, the formulations of the isotropic medium with temperature-dependent properties are proposed by the Clausius inequality and L-S generalized theory [10], where the higher order expansions of the Helmholtz free energy with respect to increment temperature are used to describe the relations of each material parameter with real temperature. The problem of a thin plate composed of titanium alloy, subjected to a sudden temperature rise at the boundary, has been conducted. The propagations of thermoelastic wave and thermal wave, as well as the distributions of displacement, temperature and stresses, which are induced by boundary thermal shock, are obtained and plotted. The comparison with the results obtained from the case of constant material properties is conducted to reveal the effect of variable material properties on thermoelastic interaction. The comparison with the results from the case that each material parameter is the function of fixed temperature is also conducted to explain the coupling effect between variable material properties and temperature distribution.

2. Formulations of the problem

Based on Clausius inequality, the strain tensor γ_{ij} and the entropy density φ can be expressed as

$$\sigma_{ij}=\frac{\partial\Psi}{\partial\gamma_{ij}}, \tag{1}$$

$$\varphi=-\frac{1}{\rho}\frac{\partial\Psi}{\partial T}, \tag{2}$$

where ρ is the mass density, T is the absolute temperature, $\Psi=\Psi\left(\gamma_{ij},\theta\right)$ is the Helmholtz free energy per unit volume, and $\theta=T-T_0$ is the increment temperature.

In order to obtain the constitutive relations of elastic medium, the power series expansion of the Helmholtz free energy $\Psi=\Psi\left(\gamma_{ij},\theta\right)$ with respect to strain tensor γ_{ij} and increment temperature θ is used to obtain the constitutive equations. As for an isotropic material, the Helmholtz free energy $\Psi=\Psi\left(\gamma_{ij},\theta\right)$ can be expanded in a power series with the following forms [25]:

$$\Psi=\Psi\left(\gamma_{ij},\theta\right)=a_0+a_1I_1+a_2I_2+a_3I_3+a_4\theta+a_5I_1^2+a_6I_1I_2+a_7I_1I_3+a_8I_1\theta+\cdots, \tag{3}$$

where $I_1=\gamma_{ii}$, $I_2=\frac{1}{2}\left(\gamma_{ii}\gamma_{jj}-\gamma_{ij}\gamma_{ji}\right)$ and $I_3=\det\gamma_{ij}$ are the invariants of strain tensors, a_0, $a_1,\cdots a_8$ are the material constants determined by experiments of material properties.

To ignore the terms of higher than the second order of θ and γ_{ij} for expansion (3), the constitutive equation based on Eq. (1) can be derived as

$$\sigma_{ij}=a_1\delta_{ij}+(a_2+2a_5)\gamma_{kk}\delta_{ij}-a_2\gamma_{ij}+a_8\theta\delta_{ij}, \tag{4}$$

where a_1 is regarded as the initial normal stress and is zero in natural state, and δ_{ij} is the Kronecker delta.

Introducing Lame's constants λ, μ and thermoelastic coupling coefficient β, then Eq. (4) can be rewritten as

$$\sigma_{ij}=\lambda\gamma_{kk}\delta_{ij}+2\mu\gamma_{ij}-\beta\theta\delta_{ij}, \tag{5}$$

where $\lambda=a_2+2a_5$, $\mu=-\frac{a_2}{2}$, and $\beta=-a_8$.

It is obvious that these material parameters are constants, and the constitutive Eqs. (4) or (5) is the governing equation with constant material properties. In order to consider the variation of material properties with temperature, the third order or even higher order terms with the increment temperature θ should be taken into account in the following derivation. Here the following expansions of $\Psi=\Psi\left(\gamma_{ij},\theta\right)$ including the third order terms of θ are used:

$$\Psi\left(\gamma_{ij},\theta\right)=a_0+a_1I_1+a_2I_2+a_4\theta+a_5I_1^2+a_8I_1\theta+a_9I_2\theta+a_{10}\theta^2 +a_{11}I_1^2\theta+a_{12}I_1\theta^2+a_{13}\theta^3. \tag{6}$$

Substituting expression (6) into Eq. (1) results in

$$\sigma_{ij}=(\lambda_0+\lambda_1\theta)\gamma_{kk}\delta_{ij}+2(\mu_0+\mu_1\theta)\gamma_{ij}-(\beta_0+\beta_1\theta)\theta\delta_{ij}, \tag{7}$$

where $\lambda_0=a_2+2a_5$, $\mu_0=-\frac{a_2}{2}$, and $\beta_0=-a_8$ are Lame's constants and thermoelastic coupling coefficient at the reference temperature T_0, which are the same forms of the case with constant material properties, $\lambda_1=a_9+2a_{11}$, $\mu_1=-\frac{a_9}{2}$ and $\beta_1=-a_{12}$ are regarded as the influence factors of temperature deviation, and a_2, $a_5,\cdots a_{12}$ are material constants determined by the real relations of the modulus elasticity and Poisson's rate with respect to temperature.

The general forms of Eq. (7) with temperature-dependent properties can be rewritten as

$$\sigma_{ij}=\lambda(\theta)\gamma_{kk}\delta_{ij}+2\mu(\theta)\gamma_{ij}-\beta(\theta)\theta\delta_{ij}. \tag{8}$$

Note that the constitutive Eq. (8) with variable material properties is nonlinear, while the constitutive Eq. (5) with constant properties is linear. However, the governing equations have the same from for two cases.

The equations of motion take the form:

$$\rho\ddot{u}_i=\rho f_i+\sigma_{ij,j}. \tag{9}$$

Substituting Eq. (8) into above motion Eq. (9), we have

$$\rho\ddot{u}_i=\rho f_i+\left[\lambda(\theta)\gamma_{kk,i}+2\mu(\theta)\gamma_{ij,j}-\beta(\theta)\theta_{,i}\right] +\left[\lambda_1\theta_{,i}\gamma_{kk}+2\mu_1\theta_{,j}\gamma_{ij}-\beta_1\theta_{,i}\theta\right], \tag{10}$$

where u_i is the displacement vector, and f_i is the body force per unit mass. Meanwhile, the superscript dot $(\cdot)$ and the subscript comma (,) denote the derivatives with respect to the time t and coordinates $x_i(i=1,2,3)$, respectively.

The energy balance equation takes the form:

$$q_{i,i}=\rho r-\rho T_0\varphi \tag{11}$$

where r is the internal heat source per unit mass, and q_i is the heat flux vector.

Substituting Eq. (2) into Eq. (11) and considering the expression (6) results in

$$q_{i,i}=\rho r-\rho c_p\dot{\theta}+T_0\left[\lambda_1\gamma_{kk}\delta_{ij}+2\mu_1\gamma_{ij}-(\beta_0+2\beta_1\theta)\delta_{ij}\right]\dot{\gamma}_{ij}, \tag{12}$$

where $c_p=-T\frac{\partial\varphi}{\partial T}=c_{p_0}+c_{p_1}\theta+c_{p_2}\theta^2$ is the specific heat, $c_{p_0}=(2a_{10}+2a_{12}\gamma_{kk})T_0$, $c_{p_1}=(2a_{10}+2a_{12}\gamma_{kk}+6a_{13}T_0)$, and $c_{p_2}=6a_{13}$. For the constant strain tensor γ_{ij}, the specific heat c_p is the quadratic function of increment temperature.

Due to the L-S theory of generalized thermoelasticity [10], the modified heat conduction equation can be expressed as

$$q_i+\tau_0\dot{q}_i=-kT_{,i}, \tag{13}$$

where k is the thermal conductivity of the isotropic medium,