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Axial crushing of tapered circular tubes with graded thickness

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ABSTRACT

This paper aims to investigate the energy absorption characteristics of tapered circular tubes with graded thickness (TCTGT) under axial loading. TCTGT specimens were fabricated by a tube tapering machine and the forming effects on crush response were investigated. Both the original straight circular tube and the fabricated TCTGT were tested and compared to analyze the relative merits of TCTGT. Numerical simulations of the tests were conducted by using nonlinear finite element code LS-DYNA and a simplified fabrication process was also simulated. The energy absorption efficiency of the fabricated TCTGTs was found to be considerably higher than that of straight tubes and the forming effects showed important influence on the increase of efficiency. In addition, a novel approach was proposed to predict the mean crushing force of circular and tapered tubes with and without forming effects. The outcomes of the present study will facilitate the design of TCTGT structures with better crashworthiness performance.

1. Introduction

Thin-walled circular or square metal tubes are known to be efficient collapsible energy absorbers under axial compression. They are extensively studied and widely applied in the engineering fields. Relatively few research efforts have been undertaken to study the energy absorption performance of tapered thin-walled structures. Actually, tapered circular or square tubes will show even better performance since they provide better designability. From the view of structural optimization, the design domain of tapered tubes is larger than that of straight tubes and therefore the crashworthiness performance of the former will definitely be better than or equal to the latter. The relatively few concerns and applications on tapered tubes should be attributed to the fact that it is relatively difficult to manufacture the tapered tubes.

The studies on the axial crushing responses of tapered thin-walled structures were initiated in the early 1980s. Experimental and theoretical analyses were carried out for tapered circular tubes (frusta) by Mamalis et al. [1–3] and tapered square tubes by Reid and Reddy [4]. In the past decade, numerical simulations on the crashworthiness of tapered tubes under axial or oblique loads attracted considerable interests [5–12]. There are primarily two advantages of tapered tubes over straight tubes: first, tapered tubes reduce the initial peak force under axial loading and generally show higher load uniformity (the ratio of mean force to initial peak force) and second they own better performance under oblique loading.

Up to now, the studies on the crushing behavior of thin-walled tubes are primarily concentrated on the structures with uniform wall thickness. Few investigations are concerned with crushing of structures with varying wall thickness. Chirwa [13] first studied the inversion buckling of taped circular tubes with variable thickness along the axial direction and reported significant efficiency increase (up to 50%) when compared to tubes with uniform thickness. Gupta [14] also studied the similar deformation behavior of metallic frusta with varying wall thickness. Recently, the crushing behavior of square tubes with graded thickness in the transverse direction was investigated by Zhang et al. [15]. Multi-objective structural optimization for square tube with graded thickness in the longitudinal direction was studied by Sun et al. [16]. Adopting graded thickness was validated by them to be an effective way to increase the energy absorption efficiency of thin-walled structures. Actually, just like employment of tapered structures, the introduction of graded wall thickness will also broaden the design domain of thin-walled structures and can definitely improve the crashworthiness performance of them. However, the fabrication of structures with graded thickness could be also a problem in the past and hampered the relevant studies and applications. As the advance in material processing technology, the manufacture of metal plates with continuous thickness changes is not difficult any more. For instance, tailor rolled blank (TRB) technology [17] is now employed to produce vehicle components with lighter weight.

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Considering the advantages of tapered structures and the better designability of structures with graded wall thickness, the crushing behavior of tapered circular tubes with graded thickness (TCTGT) under axial loads was investigated in the present work. The TCTGT specimens were first fabricated by a tube tapering machine [18] which tapered the circular tubes with uniform thickness to cone tubes with graded thickness. Axial compression tests were then carried out for the circular tubes before and after tapering and the energy absorption characteristics of them were described and compared. Numerical simulations with and without consideration of the fabrication process were conducted by using nonlinear finite element code LS-DYNA and the metal forming effects of the fabrication process on the energy absorption of TCTGT were investigated. Finally, a theoretical method was proposed to predict the flow stress and the mean crushing force of TCTGT specimens with consideration of the forming effects.

2. Efficiency of TCTGT under axial compression

Theoretically, TCTGTs are circular tubes with varied diameter in the longitudinal direction and therefore the energy absorption efficiency under axial loading can be analyzed qualitatively based on the theoretical results of circular tubes. The crushing behaviors of circular tubes under axial compression have been extensively studied and theoretical expressions were derived by researchers to predict the crush resistance of the tubes deforming in progressive buckling modes. Besides global buckling, circular tubes can develop concertina mode, diamond mode or mixed mode. The diamond mode is relatively more complicated and the theoretical models for it have not been successfully developed [19].

As for concertina mode, theoretical models were presented by Alexander [20], Abramowicz and Jones [21] and Wierzbicki et al. [22]. The mean crushing force of circular tube was derived as

$$P_m = 6.08\sigma_0 D^{0.5} t^{1.5} \text{ [20]} \tag{1}$$

$$P_m = \frac{20.79(D/t)^{0.5} + 11.9}{0.86 - 0.568(t/D)^{0.5}} M_0 \text{ [21]} \tag{2}$$

$$P_m = 7.935\sigma_0 D^{0.5} t^{1.5} \text{ [22]} \tag{3}$$

where D and t are mean diameter and wall thickness, respectively. σ_0 is the flow stress of the structural material and $M_0 = \sigma_0 t^2/4$ is the fully plastic bending moment per unit width. The theory of Alexander [20] did not consider the effective crushing distance and to some extent underestimated the mean crushing force, while Eqs. (2) and (3) give similar results. For the sake of simplicity, Eq. (3) is employed here to analyze the efficiency of circular tubes. The index SEA (specific energy absorption) of circular tube developing concertina mode can be calculated by

$$SEA = \frac{P_m \delta_e}{m} = \frac{7.935\sigma_0 D^{0.5} t^{1.5} \delta_e}{\rho \pi D t L} = \frac{7.935\sigma_0 \delta_e}{\rho \pi L}\left(\frac{t}{D}\right)^{0.5} \tag{4}$$

where ρ is the density of the structural material, L is the length of the tube and m is the mass of the tube which is given by $\rho \pi D t L$ approximately. δ_e is the effective crushing distance which is determined as $0.86 - 0.568(t/D)^{0.5}$ by Abramowicz and Jones [21]. It is only slightly dependent on the ratio of t/D since t/D is generally a small value. From Eq. (4), it can be concluded that SEA value of a circular tube increases with the increase of wall thickness and decreases with the increase of diameter. That is, the smaller the diameter and the thicker the wall thickness, the higher the energy absorption efficiency.

If the mass of a circular tube is kept constant, SEA of it can be expressed as

$$\text{SEA} = \frac{P_m \delta_e}{m} = \frac{7.935\sigma_0 \delta_e}{(\rho \pi L)^{1.5} m^{0.5}} t = \frac{7.935\sigma_0 \delta_e m^{0.5}}{(\rho \pi L)^{2.5}} D^{-1} \ (m \text{ is const.}) \tag{5}$$

Therefore, if one end of a circular tube is shrunk (diameter is reduced and thickness is increased) by the metal forming process, the SEA value will increase even without considering the forming effects on energy absorption.

Besides SEA value, load uniformity is another important index for energy absorber. For tapered tubes with uniform thickness under axial loading, the mean crushing force for the lobes in the two ends will be different due to diameter variations. Considering that the initial peak force of circular or tapered tubes can be eliminated by initial triggers or imperfections, the load uniformity of tapered tubes may be lower than that of circular tubes in this case. However, the load variations of tapered tubes between different lobes can also be eliminated by introducing graded thickness along the longitudinal direction. Assume D_1, t_1 and D_2, t_2 are the mean diameter and thickness in the two ends of a tapered tube, respectively. According to Eq. (3), the mean force of the two end lobes will keep constant if

$$D_1^{\,0.5} t_1^{\,1.5} = D_2^{\,0.5} t_2^{\,1.5} \tag{6}$$

Apparently, the thickness is not linearly distributed with the variation of diameter. Although TCTGTs with nonlinear thickness distribution can be fabricated by computer numerical control (CNC) machines, tubes with linear distribution are relatively easy to obtain. It is noted that if $0.5D_2 \le D_1 \le 2D_2$, the thickness distribution can be approximately deemed as linear. In this case, determining the thickness in two ends and making it linearly distributed along the longitudinal direction will only result in negligible fluctuation in mean crushing force.

According to the discussion above, the SEA of tapered circular tube under axial compression will lie between two circular tubes, whose diameter and thickness are the same as the two end sections of the tapered tube. This conclusion is valid no matter whether uniform thickness or graded thickness satisfying Eq. (6) is adopted. However, the results could be different if the forming effects in the fabrication of TCTGTs are considered.

3. Experimental test of circular tubes and TCTGTs

Axial compression test of circular tubes with uniform thickness and TCTGTs made of steel ST12 were carried out in the present work. The specimens of circular tubes and TCTGTs are shown in Fig. 1. The outer diameter and wall thickness of circular tubes are 60.0 and 1.50 mm, respectively. During fabrication, one end of a circular tube was inserted into the tube tapering machine [18] to form a tapered tube and the deformed tubes were then cut to TCTGT specimens with certain length. It should be mentioned that the sectional area (material) of the tubes are almost kept constant during tapering deformation. The length of the circular tubes is to some extent increased after deformation but the increase percentage is found to below 2%. The sectional views of the tapered tubes are also shown in Fig. 1. Graded thickness along the longitudinal direction can be seen in the zoom view of the section. The mean diameters in the two ends of the tapered tubes are $D_1 = 58.5$ mm and $D_2 = 46.1$ mm, while the thicknesses in these two ends are measured to be $t_1 = 1.50$ and $t_2 = 1.90$ mm. Two different lengths (different semi-apical angles) 126.0 and 200.0 mm were adopted to analyze the influence of tube length on response of the structure. The thickness distributions of the TCTGT specimens along the longitudinal direction are plotted in Fig. 2. The thickness is basically linear distributed and the maximum deviation from linear is about 1.8%. In addition, the original circular