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A two-surface hardening plasticity model based on non-associated flow rule for anisotropic metals subjected to cyclic loading

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ABSTRACT

In this paper a phenomenological material model for simulation of sheet metal forming processes was introduced. This model is able to describe the anisotropic behavior of sheet metals in both yield stresses and plastic strain ratios (*r*-values) by using the non-associated flow rule and quadratic yield and potential functions. Additionally, to reproduce an accurate prediction of cyclic plastic deformation phenomena, a two-surface mixed isotropic-nonlinear kinematic hardening model was combined with the quadratic non-associated anisotropic formulation. This mixed kinematic hardening model is amongst the most sophisticated models with acceptable degree of complexity and minimum requirement of experimental tests for material coefficients. The main advantage of the model over the more complex nonquadratic yield and potential functions along with associated or non-associated flow rules is its simplicity and computational efficiency. The plasticity foundation of the model was introduced and then a general return mapping algorithm for numerical stress integration of the constitutive model was developed in order to implement it into a finite element code. Finally, the model was used to simulate both forming stage and subsequent springback of a deep drawing problem. The results showed that the model can accurately predict springback as well as earing phenomenon of the stamped part.

1. Introduction

Anisotropy or directional dependency of material properties is usually an evident phenomenon in sheet metals. The origin of anisotropy is mainly the formation of crystallographic texture or preferred orientation of grains during rolling process [1]. During sheet metal forming processes, anisotropy of flow stress and plastic strain affects the process and product parameters such as earing of formed parts, forming forces, sheet formability, etc. While anisotropy is mainly related to microstructural features of metals and alloys, most of mathematical models to constitute anisotropic behavior have been proposed based on continuum yield functions. Many of these phenomenological models were originally suggested and developed for more effective description of different types of sheet metal anisotropy.

Hill pioneered a quadratic anisotropic yield function based on the von Mises yield criterion that is so-called Hill'48 model [2]. This model has extensively been used for more than half of a century in different applications. Despite of popularity, the Hill'48 model has some shortcomings such as, so-called anomalous behavior for many aluminum alloys. Another deficiency of this model is the fact that anisotropy of yield stress and plastic strain cannot be taken into account at the same time. To overcome these shortcomings and promote the accuracy of anisotropic yield functions for sheet metal forming simulations, several advanced models have been developed in recent decades [3–10]. It should be noted that most of these models are based on associated flow rule (AFR) and non-quadratic formulation which makes them lengthier and more expensive than quadratic models with respect to experimental requirements for material constants as well as numerical computations.

With a different point of view, Lademo et al. [11], based on experimental investigations, recommended the use of non-associated flow rule (NAFR) along with some classical anisotropic functions as yield and potential for more accurate description of anisotropy of yield stress and plastic strain by the same model. Stoughton [12] proposed a non-associated plasticity model for sheet metals based on Hill'48 function. To generalize the applicability of the quadratic model two distinct functions were defined by *r*-values for potential function and by yield stresses for yield function. After this research, a series of papers by Stoughton and Yoon [13,14] published on expansion and consolidation of the idea of using NAFR for different applications in metal plasticity. Furthermore, other researchers used NAFR theory for better description of plastic deformation and damage behavior of different metals and alloys under various loading conditions [15–18]. Their results revealed that using NAFR

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along with quadratic and non-quadratic yield and potential functions could improve the accuracy of the predictive plasticity models.

Recently, there have been various researches on numerical algorithms for finite element implementation of different types of anisotropic NAFR material models. Cvitanic et al. [19] proposed a numerical integration algorithm for anisotropic NAFR model with isotropic hardening based on a multi-stage return mapping algorithm and implemented the model for finite element simulation of sheet metal forming. Coombs and Crouch [20] developed a stress integration algorithm based on an analytical backward Euler method for a pressure-sensitive yield criterion based on a modified Reuleaux triangle using a volumetrically non-associated flow rule. Li [21] developed a nonlinear purely kinematic model with NAFR for kinematic shakedown analysis which can be used for both isotropic and anisotropic materials. Taherizadeh et al. [22,23] developed and numerically implemented more advanced models based on NAFR for quadratic and non-quadratic yield and potential functions combined with mixed isotropic-nonlinear kinematic hardening laws. They showed that a significantly better agreement with experimental results was obtained by using the NAFR and mixed hardening models for simulation of springback and earing of different sheet metals. A similar research was conducted by Safaei et al. [24] to implement the combination of non-quadratic yield and potential functions with NAFR and a modified Chaboche mixed nonlinear hardening model. They used a one-surface cyclic hardening model to predict the Bauschinger effect, transient behavior and permanent softening. Park and Chung [25] developed a NAFR formulation with combined isotropic–kinematic hardening law which leads to a symmetric stiffness modulus unlike the standard derivation procedure which results in a non-symmetric stiffness modulus. They used non-quadratic yield and potential functions and simulated the earing formation of different Al-alloys. Safaei et al. [26] studied the conditions at which the definition of equivalent plastic strain in NAFR approach can be simplified so that the numerical implementation scheme would be more convenient without loss of accuracy. Also, they used a fully implicit return mapping scheme for stress integration of the proposed constitutive model.

Usually, sheet metals undergo cyclic deformation and reverse loading paths during the manufacturing processes of different kinds of products. It is well-known that cyclic deformation significantly affects the plastic yielding, flow and hardening behavior of sheet metals which can influence the product and process parameters such as, springback, residual stresses, die design, etc. To capture these cyclic effects a very common approach is using kinematic hardening models based on backstress concept along with conventional isotropic hardening in constitutive models which are usually called combined or mixed hardening models. Many efforts have been put forth on research to develop, propose and use different kinds of advanced one-surface or multiple-surface nonlinear kinematic hardening rules for sheet metals during last decades [27–36]. Many of these models were formulated with inspirations from nonlinear Armstrong–Frederick–Chaboche and multi-surface kinematic hardening models for different types of quadratic and non-quadratic anisotropic yield functions. A review of these classical kinematic hardening models was presented by Chaboche [37]. The main challenge for these constitutive models is to capture as many cyclic phenomena as possible during cyclic plastic deformation such as, Bauschinger effect, transient elasto-plastic behavior, permanent softening, work hardening stagnation, etc. At the same time, it is vital for advanced models to keep it as simple as possible to save costs for numerical computations and experimental procedures for material coefficients determination. Although some of these models are able to predict the cyclic behavior of sheet metals with rather enough accuracy, they are formulaically complex to implement and also they need laborious and expensive experimental tests to find the material coefficients.

Yoshida and Uemori [33,35] proposed a two-surface plasticity model for large strain cyclic elasto-plasticity that is able to reproduce many cyclic deformation phenomena such as transient Bauschinger effect, permanent softening and work hardening stagnation. This model is based on the two-surface plasticity model with a new equation for the backstress evolution of the internal yield surface within a bounding surface with specified mixed isotropic–kinematic hardening model. The capability and accuracy of this model have been demonstrated in different ranges of plastic strain as well as various cyclic deformation regimes based on different experimental and simulation studies [31,32,35,38]. Another advantage of the Yoshida–Uemori model is the number of material coefficients and experimental tests required for the model calibration which is smaller than that of other advanced mixed isotropic–kinematic hardening models. This model was modified and used to develop a plasticity model in this study.

In this paper, a new comprehensive material model for simulation of sheet metal forming is introduced. The model is a combination of quadratic non-associated flow rule for describing the anisotropic behavior of material in both yield stresses and *r*-values and the modified Yoshida–Uemori two-surface mixed isotropic-nonlinear kinematic hardening model for reproducing the cyclic phenomena during cyclical plastic deformation. The two parts of the proposed model are amongst the most sophisticated constitutive laws for different sheet metal forming processes simulations with the least complexities compared to other advanced models. At first, the plasticity foundation of the model was introduced. In order to implement the model into a finite element program, the computational integration of the model to update all variables in each time increment must be derived. A general return mapping algorithm for updating of stress, plastic strain and all internal variables was developed. In order to improve the stress integration procedure in comparison with Euler backward integration method, a semi-analytical integration procedure was used to integrate the backstress equations. For model verification, different experimental procedures were designed and conducted. Finally, the model was used to simulate both the forming stage and subsequent springback of a deep drawn part.

2. Constitutive model

The following constitutive model is presented in a co-rotational coordinate system in which the coordinate system rotates with the material. This assumption makes the derivation of equations and its numerical implementations more convenient. It is also noted that the commercial finite element packages usually require the user to update the stress and internal variables in a co-rotational coordinate system. When writing a user defined material subroutine in ABAQUS, all components of stress and strain are defined and stored in the material coordinate system automatically.

The present constitutive model of plasticity contains three surfaces: a) yield surface, b) bounding surface, and c) plastic potential surface. The yield surface determines the elastic domain in the stress space. The bounding surface controls the motion of the yield surface and represents the global workhardening of the material. Finally, the plastic potential surface is used to identify the direction of plastic strain increment. These surfaces are defined as follows:

$$f_y(\boldsymbol{\sigma}-\boldsymbol{\alpha})-y=0 \tag{1}$$

$$F(\boldsymbol{\Sigma}-\boldsymbol{\alpha}_2)-b_0-r(p)=0 \tag{2}$$

$$f_p(\boldsymbol{\sigma}-\boldsymbol{\alpha})-y_p=0 \tag{3}$$

where f_y is the yield function, F the bounding surface function, f_p the plastic potential function, $\boldsymbol{\sigma}$ the stress tensor, $\boldsymbol{\alpha}$ the backstress tensor,