International Journal of Mechanical Sciences 92 (2015) 146–153

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Size effects on nanomechanical behaviors of nanoelectronics devices based on consistent couple-stress theory

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ARTICLE INFO

Article history:
Received 10 June 2014
Received in revised form
13 November 2014
Accepted 13 December 2014
Available online 19 December 2014

Keywords:
Carbon nanotube
Couple stress theory
Electromechanical behaviors
Nanoelectronics devices

ABSTRACT

The paper presents the size-dependant mechanical behavior of the carbon nanotube-based nano-electromechanical systems using the consistent couple-stress theory. The formulation of the Euler–Bernoulli beam model applied in mechanical modeling of the carbon nanotubes is developed via considering true continuum kinematical displacement and rotation. This is done to solve the indeterminacy of the spherical part of the couple-stress tensor and the appearance of the body couple in the constitutive equation of the force–stress tensor of the original coupe stress theory. The governing equations are solved in order to obtain the size-dependent amount of deflection and pull-in voltages of the carbon nanotubes under electrostatic actuation. The results are provided for various dimensions and boundary conditions. Moreover, the influences of couples-stress theory on the outcomes are scrutinized. The comparison between the results obtained from the classical and couple stress theory reveals that application of the latter leads to a model of the CNT with higher stiffness, smaller deflection and larger pull-in voltages.

1. Introduction

Micro- and nanosystems, in general, and their electromechanical counterparts (MEMS/NEMS), in particular, have found tremendous growth, recently [1–6]. They have many potential and actual applications in various high-tech industries such as micro/nano-sensors, actuators, transistors, switches, capacitors, resonators, random access memories, and filters. They can be applied in integrated-circuit (IC) technology to help for miniaturization of the ICs more and more. They are generally micro/nano-structures with various shapes combined with electrical sensing and/or actuation mechanisms. Various materials are applied in the MEMS/NEMS technologies and obviously all of them have their own pros and cons. However, among all substances, silicon-based materials are nowadays extensively utilized in electronic chips, in general, and micro/nano electronic circuits, in particular. But, one of the main disadvantages of the silicon is energy dissipation. In fact, silicon-based elements with extra-tiny sizes result in energy losing and thermal production. Thus, other materials should be replaced in the micro and nano-scales. Carbon-based nanomaterials such as carbon nanotubes (CNTs) and graphene sheets may be better choices. The electronic chips and elements produced from these materials may have higher speed, more effectiveness and smaller sizes. In addition, the problem of energy dissipation is resolved effectively. Thus, carbon nanomaterials are known nowadays as an applicable candidate in micro and nano-electronics.

The growing application of MEMS and NEMS makes the researchers to work hard on their design, analysis, characterization and fabrication. Since the basic concept of their work is mechanical defection, precise models are required to analyze their mechanical behavior. Some experiments revealed that the micro and nano-structures show higher stiffnesses in comparison to the corresponding bulk materials [7,8]. Thus, classical elasticity theories (CETs) do not yield results possessing good agreements with the experimental data. For this reason, scientists tried to develop some novel theories to compensate the shortages of the CETs [8–12]. One of the main theories for this purpose is the couple stress theory (CST) presented and developed by Cosserat, Toupin, Mindlin, Tiersten and Koiter [8,9,13]. The original CST had shortages in some aspects. Perhaps the most disturbing difficulties regarding the original CST are the indeterminacy of the spherical part of the couple-stress tensor and the appearance of the body couple in the constitutive equation of the force–stress tensor [8,14]. This inconsistent theory is called the indeterminate couple stress theory in the literature [15]. Due to the mentioned inconsistency, a number of alternative theories were developed such as the modified couple stress theory, strain gradient theory, etc [10,11]. However, none of them could compensate the shortages related to the CST effectively and some critiques were mentioned about them [14]. Recently, Hadjesfandiari and Dargush presented a novel consistent formulation for the CST via considering true continuum kinematical displacement and rotation which resolves

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http://dx.doi.org/10.1016/j.ijmecsci.2014.12.009
0020-7403/

the problems associated with the original CST in their research works [14,16,17].

One of the main applications of the CNTs is in the NEMS. The most effective actuation technique to actuate the CNT-based nano-electronic devices is the electrostatic actuation mechanism. Some researchers previously studied the electromechanical behaviors of the electrostatically actuated CNTs [18–23]. However, none of them considered the CST in their investigations. The fact is that the molecular dynamics results proved that the CNTs behave size-dependently during the electrostatic actuation [24]. Thus, the reported results computed using the CETs may have deviations from the experimental data. Thus, in this paper, we are going to apply the newly developed CST to analyze the electromechanical behaviors of the CNTs under the electrostatic actuation.

2. System description

According to Fig. 1, the CNT as the movable electrode is suspended over some graphene sheets as the fixed electrode or ground plate. When an electrical potential difference is applied on the electrodes, the charge distribution over them results in deflecting the CNT toward the ground plate. If the applied voltage increases from a critical value, the CNT cannot tolerate the attractive force resulted from the electrostatic voltage ad vdW interactions and suddenly drops down on the graphene sheets. This phenomenon is known as the pull-in instability and the associated threshold voltage is the pull-in voltage. This paper is to investigate this electromechanical behavior for various dimensions and boundary conditions using the CST.

3. Mathematical formulation

In the size-dependant couple-stress theory, the equations of equilibrium are formulated as [14]:

$$\sigma_{ji,j}+F_i=0 \tag{1}$$

$$\mu_{ji,j}+\varepsilon_{ijk}\sigma_{jk}=0 \tag{2}$$

where σ_{ji} and μ_{ji} respectively represent the force- (classical) and couple-stress tensors. In addition, F_i and ε_{ijk} denote the body force per unit volume and permutation or Levi-Civita symbol, respectively. Hadjesfandiari and Dargush proved that in the couple-stress theory, the body force and body couple are not distinguishable from each other and the body couple transform to the equivalent body force [14]. Moreover, in the couple-stress theory, unlike the classical elasticity, the stress tensor is generally non-symmetric. Thus, it can be decomposed to the symmetric and skew-symmetric components as following:

$$\sigma_{ji}=\sigma_{(ji)}+\sigma_{[ji]} \tag{3}$$

where $\sigma_{(ji)}$ is the symmetric part and $\sigma_{[ji]}$ is the skew-symmetric part of the force–stress tensor. The former will be presented in Eq. (12) and the latter will be formulated in Eq. (15). Unlike the force–stress tensor, as discussed later, the couple stress tensor will have only skew-symmetric part.

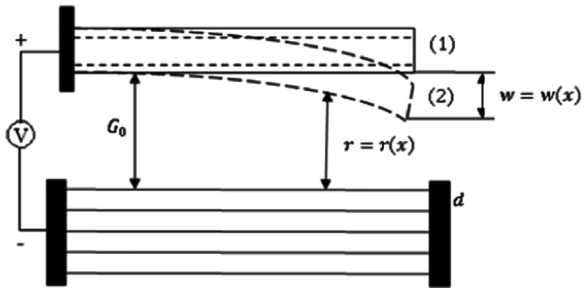

Fig. 1. Schematic view of electrostatic actuation of a CNT.

In order to define the elements of Eqs. (1)–(3) required in the couple-stress theory, the kinematic parameters should be utilized. The displacement gradient can be decomposed into two distinct parts:

$$u_{i,j}=e_{ij}+\omega_{ij} \tag{4}$$

where

$$e_{ij}=u_{(i,j)}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{5}$$

$$\omega_{ij}=u_{[i,j]}=\frac{1}{2}(u_{i,j}-u_{j,i}) \tag{6}$$

In the above relations, e_{ij} and ω_{ij} are, strain and rotations tensors, respectively. Similar to the couple-stress tensor, the rotation tensor is skew-symmetrical and a vector can be defined dual to it as below:

$$\omega_i=\frac{1}{2}\varepsilon_{ijk}\omega_{kj} \tag{7}$$

The gradient of rotation tensor can be decomposed to two sub-tensors as:

$$\omega_{i,j}=\chi_{ij}+\kappa_{ij} \tag{8}$$

where

$$\chi_{ij}=\omega_{(i,j)}=\frac{1}{2}(\omega_{i,j}+\omega_{j,i}) \tag{9}$$

$$\kappa_{ij}=\omega_{[i,j]}=\frac{1}{2}(\omega_{i,j}-\omega_{j,i}) \tag{10}$$

The diagonal terms in χ_{ij} represent the pure torsion about the coordinate axis and the off-diagonal terms are deviations from sphericity [14,25]. It does not contribute as a fundamental measure of deformation and will not be included in the strain energy [14]. On the other hand, in the couple-stress theory, the curvature tensor (κ_{ij}) plays a crucial role in the strain energy. The corresponding dual vector of the skew-symmetric curvature tensor can be formulated as:

$$\kappa_i=\frac{1}{2}\varepsilon_{ijk}\kappa_{kj} \tag{11}$$

The constitutive relations for isotropic materials are as follows. The symmetrical part of the force–stress tensor in Eq. (3) is same as the force–stress tensor in classical elasticity and can be obtained from Eq. (12).

$$\sigma_{(ji)}=\lambda e_{kk}+2\mu e_{ij} \tag{12}$$

where λ and μ are the Lame's constants. The couple-stress tensor is skew-symmetrical $(\mu_{ij}=-\mu_{ji})$ and a vector μ_i can be introduced dual to the tensor.

$$\mu_i=\frac{1}{2}\varepsilon_{ijk}\mu_{kj} \tag{13}$$

For the isotropic linear materials, Hadjesfandiari and Dargush proved that the couple stress can be computed from Eq. (14) [14].

$$\mu_i=-8\eta\kappa_i \tag{14}$$

The above relation shows that the couple-stress theory for the isotropic linear materials has only one extra size-dependant parameter. The ratio $\eta=\mu l^2$ is the constant that makes the difference between classical and couple-stress theories. The size-dependant parameter, l, varies from one material to another or from one scale to another scale. For the zero value for this parameter, the latter reduces to the former. This parameter should be obtained via experiment for various dimensions and working conditions. Also, it can be approximated by more accurate techniques such as molecular dynamics simulation. However, some technical problems are there for this purpose such as selecting and tuning the molecular dynamics parameters including the potentials for various interactions, type of