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On the algebraic parameter identification of vibrating mechanical systems

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ABSTRACT

This paper proposes an on-line algebraic parameter identification method in time domain for multiple degrees-of-freedom linear mass–spring–damper mechanical systems using position measurements. The constant parameters of mass, stiffness and damping are algebraically estimated from transient real-time measurements. Analytical and numerical results prove the accuracy and efficiency of the proposed identification approach for reference trajectory tracking control tasks on multiple-input–multiple-output vibrating mechanical systems with unknown constant parameters.

1. Introduction

Parameter identification in vibrating mechanical systems is fundamental during their modelling and experimental validation from input–output measurements. In experimental modal analysis this task is typically performed by exciting the mechanical system with an impact hammer or shaker and obtaining transient and frequency responses from force and vibration transducers and signal analyzers [1]. In the literature there exist numerous valuable identification methods to get a time or frequency domain model, mainly the case of modal parameter estimation for mechanical system (see, e.g., [1–9] and references therein). Nevertheless, most of the identification algorithms are off-line, asymptotic, recursive and slow for real-time implementations of vibration control or structural health monitoring of in-service mechanical structures [10–12].

On the other hand, a theoretical framework for algebraic parameter identification for continuous-time constant linear systems has been introduced recently in [13]. From a theoretical point of view, robustness of the algebraic identification schemes against structured perturbations and noise has been discussed and proved in [13,14]. Structured perturbations are annihilated by suitable linear differential operators. Noise is assumed to be a high frequency oscillating time function, which can be attenuated by low-pass filters as iterated time integrals. Thus, the filtering stage may be constituted by a pure integration chain of certain finite length of noisy measurement signals, avoiding the knowledge about probabilistic and statistical properties of the corrupting noise [14,15]. Indeed, algebraic parameter identification has also been proposed for on-line estimation and rejection of disturbances [16,17], and algebraic numerical differentiation of time signals in noisy environments based on truncated Taylor polynomial expansion of the signal into a quite short time window [18,19].

The algebraic identification approach is based on well-known powerful mathematical tools as module theory, differential algebra and operational calculus. Operational calculus is a quite general approach based on different integral transformations of functions and generalized functions (e.g., Fourier, Laplace, Stieltjes, Hilbert, Bessel) [20,21]. In mechanics is quite common the application of operational calculus in the transformation of functions from time to frequency domain to solve differential equations or signal processing tasks.

Algebraic identification has been proposed by the authors for on-line estimation of parameters and signals for active vibration absorption in vibrating mechanical systems [22], active unbalance control in rotating machinery [23] as well as modal parameter estimation of free flexible mechanical structures [24]. In these studies, some outstanding properties of the algebraic identification approach for vibrating mechanical systems, as accuracy, fastness and robustness, above those reported in the literature have been described numerically and experimentally.

This paper deals with the parameter identification problem in multiple degrees-of-freedom continuous-time linear mass–spring–damper systems with lumped parameters. A wide variety of vibrating mechanical systems can be approximately characterized using several topological configurations of linear mass–spring–damper systems under certain operating conditions. Some practical examples of these systems can be found in dynamic vibration absorbers [25,26],

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vibration isolation [27], rotating machinery [28], tuned particle damper [29] and probes of atomic force microscopes [30].

We propose a novel on-line algebraic parameter identification method in time domain for multiple degrees-of-freedom linear mass–spring–damper mechanical systems using position measurements. Here, the constant parameters of mass, stiffness and viscous damping of interest are algebraically and simultaneously estimated during transient system behavior and without singularity problems (algebraic persisting excitation property). The proposed identification method is used in active regulation and tracking control tasks on multiple-input–multiple-output (MIMO) vibrating mechanical systems with unknown parameters. Thus, a novel adaptive-like output feedback control scheme for MIMO mass–spring–damper systems is also introduced in this work to evaluate the performance of the presented method.

The proposed control scheme for tracking tasks of reference position trajectories planned for the mechanical system exploits the property of differential flatness exhibited by the system. A dynamical system is a differentially flat system if there exists a set of independent outputs, called flat outputs, equal in number to the control inputs, which completely parameterizes every state variable and control input [31]. By means of differential flatness the analysis and design of the tracking control scheme is greatly simplified. Differential flatness is also combined with integral reconstruction of the state vector [32,33] for the synthesis of the control scheme, which only requires position measurements, avoiding the use of conventional asymptotic state observers. Some numerical simulations are included to show the efficiency and accuracy of the proposed on-line identification method for the mass, stiffness and damping parameter estimation as well as the effectiveness of the adaptive-like output feedback trajectory tracking control scheme on a 6 degrees-of-freedom (DOF) and Multiple-Input–Multiple-Output (MIMO) vibrating mechanical system, with unknown parameters. The parameter identification and control scheme are also tested under additive stochastic noise contamination on the measurement and control signals, leading to satisfactory results.

2. Mass–spring–damper system of *n* degrees of freedom

2.1. Mathematical model

Consider the n DOF linear vibrating mechanical system schematically described in Fig. 1. The generalized coordinates are the n positions of the mass carriages, x_i, $i=1,2,\dots,n$. In addition, u_i represent the force control inputs, and m_i, k_i and c_i denote mass, stiffness and viscous damping associated to the i-th DOF.

The mathematical model of this flexible mechanical system can be also described by the coupled ordinary differential equations

$$\begin{aligned}
&m_1\ddot{x}_1+c_1\dot{x}_1+k_1x_1+k_2(x_1-x_2)=u_1\\
&m_2\ddot{x}_2+c_2\dot{x}_2+k_2(x_2-x_1)+k_3(x_2-x_3)=u_2\\
&\vdots\\
&m_{n-1}\ddot{x}_{n-1}+c_{n-1}\dot{x}_{n-1}+k_{n-1}(x_{n-1}-x_{n-2})+k_n(x_{n-1}-x_n)=u_{n-1}\\
&m_n\ddot{x}_n+c_n\dot{x}_n+k_n(x_n-x_{n-1})+k_{n+1}x_n=u_n
\end{aligned}\tag{1}$$

In more compact form the mechanical system (1) is traditionally expressed as follows:

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{u},\quad \mathbf{x},\mathbf{u}\in\mathbf{R}^n \tag{2}$$

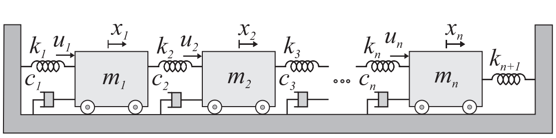

Fig. 1. Schematic diagram of a n DOF mass–spring–damper system.

where $\mathbf{x}$ is the displacement vector, $\mathbf{u}$ is an exogenous force vector, and $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are symmetric inertia, damping and stiffness $n\times n$ matrices, respectively. It is easy to verify that the system (1) is completely controllable and observable, and stable when $\mathbf{K}$ is positive definite and $\mathbf{C}\equiv\mathbf{0}$ and $\mathbf{u}\equiv\mathbf{0}$, and asymptotically stable when $\mathbf{C}$ is positive definite [34].

The configuration of the mechanical system (1) is assumed without any loss of generality and for simplicity to describe the parameter identification and control methodologies. However, the analysis can be realized on general n-DOF linear mechanical systems, which under certain conditions (e.g., using modal analysis techniques) can be transformed into a set of simple and uncoupled one-degree-of-freedom linear mass–spring–damper systems.

2.2. Differential flatness

The MIMO vibrating mechanical system (1) exhibits the differential flatness property. In fact, the flat outputs are given by the positions of the mass carriages, $y_i=x_i$, $i=1,2,\dots,n$. Hence, all the state variables, x_i and $\dot{x}_i$, and the control inputs, u_i, can be parameterized in terms of the flat outputs y_i and a finite number of their time derivatives [31]. Notice that, y_i also denotes the available position output signals of the vibrating mechanical system to be used in the parameter identification and control implementation.

From mathematical model (1), the differential parameterization results as follows:

$$\begin{aligned}
&y_i=x_i\\
&\dot{y}_i=\dot{x}_i\\
&u_i=m_i\ddot{y}_i+c_i\dot{y}_i+k_i(y_i-y_{i-1})+k_{i+1}(y_i-y_{i+1}),\quad i=1,2,\dots,n
\end{aligned}\tag{3}$$

with $y_0=y_{n+1}=0$. Hence, one obtains the following differential flatness-based controllers for tracking tasks of some position reference trajectories $y_i^*(t)$ specified for the vibrating mechanical system:

$$u_i=m_iv_i+c_i\dot{y}_i+k_i(y_i-y_{i-1})+k_{i+1}(y_i-y_{i+1}),\quad i=1,2,\dots,n \tag{4}$$

where v_i are auxiliary control inputs, synthesized to get the desired asymptotic output tracking of the reference positions y_i^*, given by

$$v_i=\ddot{y}_i^*-\beta_{1,i}(\dot{y}_i-\dot{y}_i^*)-\beta_{0,i}(y_i-y_i^*),\quad i=1,2,\dots,n \tag{5}$$

where $\beta_{0,i}$ and $\beta_{1,i}$ are parameters selected to guarantee the asymptotic output tracking with a prescribed closed-loop dynamic performance. Therefore, the closed-loop tracking error dynamics, $e_i=y_i-y_i^*(t)$, is governed by

$$\ddot{e}_i+\beta_{1,i}\dot{e}_i+\beta_{0,i}e_i=0,\quad i=1,2,\dots,n \tag{6}$$

which is globally asymptotically stable for properly design parameters $\beta_{j,i}$, j=0,1.

The differential flatness control (4) represents a solution to the tracking control problem of closed-loop position reference trajectories for MIMO vibrating mechanical systems with n-DOF. However, this control scheme requires velocity and position measurements, increasing the implementation costs and, likely, deterioration of its performance due to differentiation of position signals with respect to time. Thus, an output feedback control scheme based on differential flatness and integral reconstruction of the velocity signals is introduced in the next section using measurements of the position output variables only. The control scheme is then combined with the on-line estimation method of mass, stiffness and damping parameters for n-DOF vibrating mechanical systems proposed in this work.