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Experimental and numerical investigation of impacting cantilever beams: Second mode response

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ABSTRACT

The dynamics of a cantilever beam undergoing vibro-impact, near its second linear natural frequency, is the focus of this paper. The flexible beam undergoes vibro-impact with two other flexible beams at its free end. Both experimental and computational methods are used for the study. From this it is shown that the computational model is able to capture the experimental vibro-impact behavior very well when the excitation amplitudes are reasonably high. Further, the presence of chaos through the period doubling route is demonstrated in the experimental studies. By using independent acceleration and velocity measurements, it is demonstrated that smoothing filters suggested in the literature for generating derivatives (acceleration from velocity) does not work very well for vibro-impact problems.

1. Introduction

Vibro-impacts caused by clearances occur in several mechanical systems. For instance, gear backlash, root tip clearance of turbine blades, loose bolts and rotor–stator rubbing lead to larger vibration and noise amplitudes due to vibro-impacts. Fortunately, in a number of situations, such as rotor–stator rubbing in gas turbine engines, the use of squeeze film dampers in rotor-bearing systems or interface friction in turbine blades, the non-linear force generating elements are spatially localized. The general practice in industry is to use finite element models for analysis of such systems. These computational models are usually at least a few thousand degrees-of-freedom (DOF) in size. Due to the spatial localization of the non-linear elements, the number of DOF associated with them is significantly smaller. Therefore, for these problems, it does not make sense to use traditional non-linear solution techniques (usually numerical integration) as they demand significant computational resources.

A number of papers are available in the literature dealing with vibro-impact problems; most of these are numerical studies based on models with single or two DOF [1–8]. Some of these papers propose better algorithms for solving the differential equations, enabling the solution to pass through strongly non-linear regions without numerical difficulties. The non-linear dynamics of a cantilever beam with an end mass, excited by a periodic force, was examined by Lin et al. [2]. They developed a single DOF model, with the end mass being the primary inertia term and the beam stiffness calculated based on both contact and geometric non-linearity; both single-sided and double-sided impacts were investigated. Blazejczyk-Okolewska et al. [6] developed reduced order single and two DOF models from finite element models, for periodic motion of impacting beams. The focus was on finding appropriate parameters for the reduced models as well as the limitations associated with such an approximation. Padmanabhan and Singh [7] proposed a shooting based parametric continuation method to solve for the steady-state global dynamics of a two-degree-of-freedom piecewise non-linear system with backlash/multi-valued springs and impact damping. Using the incremental harmonic balance method (HBM), Raghothama and Narayanan [8] investigated the periodic response of a non-linear geared rotor dynamic system. An arc length continuation method for tracing bifurcation diagrams was implemented with the incremental HBM.

Several investigators have studied the vibro-impact problem of a cantilever beam impacting a stop. Yin et al. [9] investigated the behavior of a cantilever beam, driven by a periodic force, repeatedly impacting against a rod-like stop. The transient wave propagation problem was solved using a series of eigenfunctions. Wagg [10] used a collocation method instead of a Galerkin based approximation for studying the one-sided impact of a cantilever beam. He assumed a series solution for the governing equation of the beam with the spatial variation based on the linear mode shapes and coefficient of restitution model representing the impact. Knudsen and Massih [11] studied the vibration and impact dynamics of a periodically forced loosely supported beam. The beams were modelled using FEM and spring elements were used to include support flexibility. Unilateral contacts were assumed and numerical integration was used for solution. Vorst et al. [12]

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analyzed the behavior of a periodically driven beam which has an elastic stop at its middle. They used a Hertzian contact model for the contact and modelled the beam by finite elements. The size of the problem was reduced using component mode synthesis based on one, two and four modes. The solution was obtained using multiple shooting along with a path following method.

Experimental studies on vibro-impact problems are significantly smaller in number than numerical studies. Many of the available experimental studies deal with a single DOF impact oscillator. Kahraman and Blankenship [13] showed the existence of chaos in a gear pair with backlash non-linearity using carefully designed experiments. The corresponding numerical model for validation was based on a single DOF model. Rigaud and Perret-Liaudet [14] carried out an experiment on a single DOF impact oscillator with Hertzian contact stiffness. The response around the super-harmonic resonance of order 2 demonstrated the presence of an unstable transcritical bifurcation induced by the Hertzian non-linearity and a pre-existing isola corresponding to vibro-impacts. Experimental results were compared with a theoretical model based on the method of multiple scales incorporating parametric continuation. In another work by the same authors [15], they studied the effect of a pre-load on the harmonic response of the single DOF impact oscillator with Hertzian contact stiffness. When the amplitude of the harmonic excitation reaches 7% of the pre-load then the jump region (multiple solutions) extends over a fairly wide frequency range. Vorst et al. [16] are among the few investigators to have carried out vibro-impact experiments with continuous systems. They conducted experiments on a beam impacting a single-sided stop at the free end. The contact points were made of aluminum spheres whereas the beam was made of steel. A numerical model based on Hertzian theory was developed for the contact of the beam with the stop while the beam model was reduced to six DOF using component mode synthesis. The excitation amplitude was not uniform as it was based on unbalance type. Praveen Krishna and Padmanabhan [17] studied experimentally and numerically the dynamics of a flexible beam hitting two rigid stops. The system was simulated numerically using Runge–Kutta–Fehlberg (RK45) method and the results were compared with an experiment. The mode superposition method was used for reducing the model size in simulation. Ing et al. [18] carried out an experiment with a linear single DOF oscillator (mass on leaf springs) undergoing impacts with an elastic support (beam) on one side. They showed that the most common bifurcation scenario is one where a non-impacting periodic orbit bifurcates into an impacting one via a grazing orbit. This orbit was seen to be generally unstable losing its stability due to the grazing impact. Using the same experimental set-up Pavlovskaia et al. [19] have demonstrated the existence of chaotic attractors. The evolution of the attractor is shown to be influenced both by the interactions between smooth and nonsmooth bifurcations, as well as the co-existing periodic orbits.

A more complex vibro-impact problem is the rotor–stator rub interactions in an aircraft engine. Due to stricter environmental norms and increased competition, aircraft engine developers have been forced to develop higher efficiency engines, which require smaller clearances between the stator and rotor components of the engine. This increases the possibility of rubbing in engines during operation. Another cause of rub is due to the loss of one or more fan blades. Although, the failed engine is switched off, the fan continues to “windmill” as the aircraft is still flying. This causes a large mechanical unbalance excitation and the fan/compressor blades begin to rub against the stationary casing. Motivated by this, Childs [20] was among the first to study vibro-impact in a Jeffcott rotor due to bearing clearances. Using perturbation techniques, with the effects of higher harmonics included, he [21] demonstrated the presence of sub-harmonic responses. However the procedure is really cumbersome when higher harmonics are included. Choy and Padovan [22] performed a non-linear transient analysis of a simple single DOF Jeffcott rotor rubbing against a rigid casing on flexible supports. Only the rubbing phase was investigated with the non-linear effects due to loss of contact being ignored. The effect of various parameters such as support stiffness, and friction coefficient during rubbing was studied. Later, Choy et al. [23] used a multi-DOF model for rotor–stator rubbing with the blade contact being modelled using a combined longitudinal/flexural stiffness representation. Padmanabhan et al. [24] implemented a harmonic balance method based parametric continuation to investigate a large scale non-linear rotor–stator system with rubs; model reduction using component mode synthesis (CMS) enabled efficient computation of the non-linear response. Yu et al. [25] studied the effect of clearance on a rotor/seal rub both experimentally and numerically. The instability regions due to the jump are identified by them under full annular rub region.

However, the practical systems studied above are continuous systems and can get excited at higher modes depending on the operating conditions of the system. Since the practical system is quite complex, a simpler system of a beam (representing the rotor) impacting two other beams (to represent the casing) is investigated to gain insight on vibro-impact behavior at higher modes of the system. Experimental investigations of flexible beam impacts are rare [18,19] in the literature. Among those experimental investigations reported, only a few have studied two-sided impact with flexible stops and the main focus has been on the first mode response. In this paper, a cantilever beam impacting two flexible beams is investigated with the focus on the non-linear dynamics around the second beam mode. The effect of excitation (magnitude and duration) on the non-linear response is also carried out for its effect on response at higher modes as it require higher input energy to get excited.

2. Vibro-impact of flexible beams

2.1. Experimental setup

In order to study the vibro-impact of flexible beams, an experimental setup is developed. Fig. 1 shows a cantilever beam in the middle (called as the primary beam), separated from the top and bottom beams (called as secondary beams) by a clearance. The clearance is controlled with the help of a clearance spacer and is symmetric at the top and bottom. The beam dimensions have been chosen to ensure that the first few modes are flexural modes; Table 1 lists the geometric details of the beams used in the experiment with three different secondary beam lengths used. The primary beam is fixed rigidly on one end with the other end being free when assembled. Two bolts tightened to a predetermined torque, using a torque wrench, are used to maintain the clamping. This ensures that changes in boundary conditions, during the course of several experiments, are minimal.

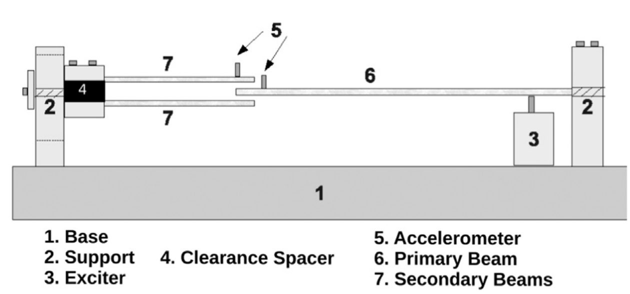

Fig. 1. Schematic of the flexible–flexible beam impact setup.