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Nonlinear dynamic and energetic characteristics of piezoelectric energy harvester with two rotatable external magnets

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ABSTRACT

This study investigated the nonlinear dynamic and energetic characteristics of an energy harvester system composed of a cantilever beam with two fully deposited piezoelectric layers and a tip magnet attached to the beam's free end. In this system, the tip magnet interacts with two external rotatable magnets fixed in free space, in which the number of equilibrium points of the tip magnet changes with respect to the inclination angle of the rotatable magnets. The field equations of the beam and magnetic force are sophisticatedly derived using the modified Hamilton's principle and magnetic current model, respectively. The derived equations are qualitatively validated by comparison with the experimental results reported in Zhou et al. (2013) [31] and by conducting experiments, it is shown that the mathematical model can predict large-amplitude motion well. The linear terms of the magnetic force and torque are properly reflected in deriving the accurate eigen-modes, and the field equation of the cantilever beam coupled with the electronic circuit is discretized into a single mode oscillator using the Galerkin method. Finally, the characteristics of the system bifurcating into a mono-, bi- or tri-stable system with respect to the inclination angle of the external magnet and separation distance between the tip magnet and external magnets are investigated with the help of both the bifurcation and potential energy diagrams. It is also shown that the broadband and power-generating characteristics of the system can be thoroughly examined by using the harmonic balance method as a simple, manipulable tool.

1. Introduction

Power supplying techniques have become a focal point in the manufacture of wireless small electronic devices over the past decade [1–6]. Much effort has been devoted to improving the performance and lifetime of a battery. However, the energy density for a battery has not reached the level required to make modern miniaturized electronic devices [1,2]. Furthermore, the maintenance cost related to replacing the battery of a sensor network such as a tire-pressure monitoring system (TPMS) [3] and the ubiquitous monitoring systems used in health monitoring, environment control, and building monitoring [4,5] is a barrier that must be overcome. Under these circumstances, the energy harvesting technique has been suggested as an alternative method to solve the problem of a limited battery life [1–26,28–32]. Applying the energy harvesting technique to a wireless electronic device provides it with a power source that has virtually an infinite lifetime without replacement. In addition, the application of this technique is achieved by reducing the power requirements needed for operating the devices using system optimization and minimizing techniques for the power dissipation of digital processors [5–7].

Among the various types of energy sources to be harvested, vibration energy has been chosen by many researchers because it exists almost everywhere, and the associated harvesting devices are, in general, cost effective and exhibit excellent power density. Most vibration energy harvesters have been a type of linear resonator, which generally has the problem of a narrow resonant bandwidth. Thus, the off-resonant frequency bands have poor performance. Over the last few years, many efforts have been made to improve the energy harvesting efficiency [8–26,28–32]. Roundy and Wright [8] and Beeby et al. [9] attempted to optimize a linear resonator-type energy harvester. Beeby et al. [9] focused on the circuit optimization of an electromagnetic type, and they concluded that the optimum condition can be achieved by using equal settings for the electromagnetic damping and parasitic damping values. Roundy and Wright [8] investigated the effects of the proof mass size, resonance frequency, mechanical damping, and load resistance on the efficiency of a cantilever beam-type energy harvester composed of two piezoelectric bending elements and a tip mass placed on the beam's free end. They then suggested some design rules for improving the harvester efficiency. Multilayer piezoelectric materials have also been suggested to increase the electric current in linear piezoelectric energy harvesters

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[10,11]. On the other hand, the development of frequency tuning techniques was another approach to resolve the poor performance issue of a linear resonator-type energy harvester, the efficiency of which drastically decreases when the ambient vibration frequency deviates from the resonance frequency of the harvester [8–26,28–30]. Leland and Wright [12] showed that the resonance frequency of a simply supported piezoelectric bimorph beam can be changed by increasing the compressive axial preload. They showed that the harvester can generate 65–90% of the nominal power at frequencies 19–24% below the unloaded resonance frequency using this tuning technique. In addition, Eichhorn et al. [13] used the tensile load for widening the frequency tuning range and Rosa and Junior [14] and Ayed et al. [15] tuned the resonance frequency by changing the shape of a beam-type energy harvester. However, these frequency tuning techniques have limitations in that they cannot automatically change the resonance frequency without an additional power supply.

To overcome the forgoing issues with vibration energy harvesters, broadband energy harvesters using nonlinear characteristics such as the stiffness softening or hardening effect have been suggested and investigated using both experimental and theoretical approaches [14–26,28–32]. Abdelkefi and Barsallo [16] investigated the cantilever-beam-type energy harvester for its use in a low-frequency excitation environment. They modeled a piezo-magnetoelastic energy harvester system together with the circuit equation and demonstrated that the softening behavior of the nonlinear system depends on the electrical load resistance. Stanton et al. [19] proposed a nonlinear energy harvester composed of a piezoelectric beam with a magnetic tip mass, which could control the stiffness softening and hardening effect by changing the configuration of four external magnets fixed in free space. Mann and Sims [21] investigated a nonlinear energy harvester that used the magnetic levitation effect, where the governing equations for the mechanical and electrical domains were derived in the form of the Duffing oscillator that resulted from the nonlinear magnetic force exerted on the center magnet by two magnets located above and below its path. Meanwhile, Soliman et al. [22] proposed a new amplitude limiter to widen the bandwidth of a linear resonator-type energy harvester, which exhibited a linear system's behavior for small amplitudes. However, when the vibration amplitude exceeded a threshold value, the amplitude limiter maintained the frequency response near the threshold value during the forward frequency sweep.

Recently, studies have been conducted on nonlinear energy harvesters that use bi-stability [17–19,23–26,28–32]. In these studies, the bi-stability could be achieved by imposing external forces on a mono-stable vibrating system possessing only one stable state at the trivial equilibrium position to make it unstable. Then, these bi-stable systems tended to have two potential energy wells formed symmetrically with respect to the previous trivial center (equilibrium) position. The mono-stable system's stable position became unstable, with a large potential energy higher than those of the surrounding positions. As a result of this potential energy structure, two types of motion could exist, i.e., intrawell and interwell motions. When the system was excited by a force with sufficient strength to jump over the potential energy formed at the center of the two potential wells, an interwell motion oscillation was generated across the wells. Because of this interwell motion, a bi-stable system could possess not only a broadband characteristic, but also a large amplitude vibration characteristic.

The bi-stable system has been investigated using various theoretical and experimental methods [17–19,23–26,28–32]. Moon and Holmes [24] proposed a new bi-stable structure known as the Moon beam, and investigated the chaotic-type non-periodic motions of the structure. They focused on the chaotic motion called "strange attractors," which exhibits a chaotic and unpredictable motion without any random parameters or inputs in the governing equations. Moreover, they found that more than two stable equilibrium positions can exist and that the chaotic motion originates from the jump motion traveling between the equilibrium positions. Their pioneering research triggered recent studies on similar types of bi-stable systems [23]. Erturk et al. [25] developed a piezoelectric energy harvester using a Moon beam structure and investigated the large-amplitude periodic oscillations and chaotic strange attractor motion. Stanton et al. [26] investigated, both theoretically and experimentally, the nonlinear and energetic characteristics of a traditional bi-stable system composed of a piezoelectric bimorph cantilever beam with a magnetic tip mass and an external magnet to impose the magnetic repulsion effect on the tip magnet. In their study, the model, which considered the discontinuity of the partially deposited piezoelectric layers, could well describe the bi-stable system. However in deriving the eigenfunctions, the linear terms included in the nonlinear magnetic force and torque were omitted, which resulted in eigenfunctions that were not accurate enough to discretize the field equation into a single mode oscillator. Thus, it could not properly reflect the instability characteristic of the axially loaded buckled beam [27]. Ferrari et al. [28] investigated the broadband energy harvesting characteristics of the simplified bi-stable model of Stanton et al. [26]. Their target system was composed of a single external magnet and ferromagnetic cantilever beam, on which two piezoelectric layers were deposited. The nonlinear and energetic characteristics of the system, which used the magnetic attraction effect, were investigated under white-noise excitation, and they demonstrated that the bi-stable system could improve the output rms voltage by up to 400% with respect to the equivalent linear resonator case. Mann and Owens [29] proposed a bi-stable system that used an electromagnetic induction mechanism for harvesting the vibration energy, and investigated the broadband characteristic of the resulting nonlinear system.

More complex energy harvesting systems composed of a cantilever beam with a tip mass and two external magnets have also been proposed [30,31], with which better energy harvesting performances could be achieved. Kim [30] showed that the system's stable state could be bifurcated into a bi- or tri-stable state by adjusting the configuration of the external magnets, and depicted the bifurcation diagrams with respect to the geometric parameters related to the configuration of these external magnets. In his study, the modified high-dimensional harmonic balance method was proposed as an efficient analysis tool, which was particularly useful for analyzing a forced vibration problem excited by a strong nonlinear magnetic force. Zhou et al. [31] also used two external magnets, and they controlled the system's stable state by simply adjusting the inclination angle of these external magnets. To investigate the broadband characteristic of the system, they developed and used a phenomenological hybrid model that could predict the system's bi-stable characteristic and its dependence on the inclination angles of the magnets. They derived electromechanically coupled equations of motion by discretizing the standard Euler–Bernoulli beam equation into its fundamental mode. However, in the discretizing step of their study, the external magnetic force term was completely omitted, and thus its linear part was not reflected in the resulting assumed single (fundamental) mode. This omission could lead to a critical error in the single mode approximation, because the modal characteristic of the system of interest should be well reflected in the single mode (mostly fundamental mode) to minimize the effect of the other modes [27].

In this study, the nonlinear dynamic and energetic characteristics of the piezoelectric energy harvester system with two rotatable external magnets proposed in [31] are analytically and