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One-equation integration algorithm of a generalized quadratic yield function with Chaboche non-linear isotropic/kinematic hardening

M. Wali *, H. Chouchene, L. Ben Said, F. Dammak

Mechanical Modelisation and Manufacturing Laboratory (LA2MP), National Engineering School of Sfax, University of Sfax, B.P W, 3038 Sfax, Tunisia

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ABSTRACT

In this paper, the implicit integration of a quadratic yield criterion exhibiting Chaboche non-linear kinematic and isotropic hardening is presented. A new expression of consistent tangent modulus is derived and implemented in finite element programs. The non-linear global equilibrium equations as well as the one single non-linear local equations obtained by fully implicit integration of the constitutive equations are solved using the Newton method. The consistent local tangent modulus is obtained by exact linearization of the algorithm. The performance of the present algorithm is demonstrated by numerical examples where a quadratic convergence behavior can be observed.

1. Introduction

Isotropic and anisotropic elastoplastic models are the appropriate choices which define the ductile metallic material behavior. In addition, efficiency and accuracy of finite element simulations of elastoplastic behavior are very important because much of the computational cost is due to the stress point integration algorithms and the predicted behavior depends completely on the material constitutive relations, [1]. Also, when studying the rate-independent elastoplasticity problems, the notion of consistency between the tangent operator and the integration algorithm plays a crucial role in preserving the quadratic rate of asymptotic convergence of iterative solution schemes based upon Newton's method, [2]. Many authors are interested in these numerical research fields. They resolved, with different methods, the complex elastoplastic constitutive equations exhibiting non-linear isotropic/kinematic hardening.

Within the framework of J_2 elastoplasticity, a consistent return mapping algorithm for 3D and plane stress are developed in Simo and Taylor [2,3]. The consistent elastoplastic tangent moduli are obtained by exact linearization of the algorithm. Afterward, an algorithm for implementation of combined non-linear isotropic/kinematic hardening, where the later is formulated due to Armstrong and Frederick [4], has been proposed by Doghri [5]. Here, the plastic multiplier and a reduced stress tensor appear as unknowns in the resulting problem. It follows that the number of unknowns is five or seven for two or three dimensional problems, respectively. An alternative formulation is presented first in Hartmann and Haupt [6] and second in Mahnken [7], where the resulting problem is reduced to one equation, such that only the plastic multiplier appears as an unknown. Also, the J_2 plasticity combined with kinematic hardening model of Lemaitre and Chaboche [8] is considered in the work of Halama and Poruba [9] to establish the tangent modulus in numerical integration and its influence on the convergence of Newton–Raphson Method. Halama et al. [10] presented a numerical implementation of the cyclic plasticity model based on the kinematic hardening rule of AbdelKarim and Ohno [11] and its further application in fatigue life prediction.

Sheet metals generally present a significant anisotropy due to their crystallographic texture. This anisotropy is the most important aspect that should be taken into account when modeling inelastic materials behavior. In addition, most engineering structures are subjected to cyclic loading which leads to additional effects of Bauschinger. Many advanced approaches have been developed to describe plastic anisotropy for an accurate modeling of the experimental behavior. The most widely used constitutive model is the one based on the classical Hill'48 yield, proposed as a generalization of the J_2 plasticity yield function for anisotropic materials.

Dutko et al. [12] proposed a stress update algorithm for the Barlat anisotropic yield criterion existing in Barlat et al. [13]. Kinematic hardening is not considered in this algorithm where the plastic multiplier and a reduced stress tensor appear as unknowns. Boubakar et al. [14] defined a stress-computation algorithm taking into account the anisotropic plastic behavior and the update of the orthotropic frame directions within a finite-element simulation of sheet-metal forming. Oliveira et al. [15] studied the influence of work hardening models in springback simulation of Numisheet'05 Benchmark #3, considering dual phase material (DP600). The Hill'48 yield criterion is used and the unknowns in the resulting problem are the plastic multiplier and a reduced stress tensor. Kim et al. [16]

* Corresponding author.
E-mail address: mondherwali@yahoo.fr (M. Wali).

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developed elastoplastic constitutive equations for highly anisotropic and asymmetric materials. In this contribution, the Drucker–Prager yield criterion is used and modified by adding anisotropic parameters and initial components of translation. In Cardoso and Yoon [17], the backward-Euler method are combined with a non-quadratic anisotropic yield functions to predict accurately the behavior of aluminum alloy sheets for sheet metal forming processes. A large number of unknowns, 12 at least, appear in the resulting problem. In order to characterize the material coefficients, including the Bauschinger ratio for the kinematic hardening model, one element tension-compression simulations were tried based on a polycrystal plasticity approach. The developed model was applied for a springback prediction of the NUMISHEET'93 2D draw bend benchmark example.

A modification of the isotropic hardening model is adopted in Chun et al. [18,19] to predict accurate hardening under cyclic loading conditions. The model is implemented into a user-defined material subroutine (UMAT) into ABAQUS/Standard code based on the fully implicit backward Euler's method. The modified isotropic hardening of Chun et al. [18] is adopted in Safaei et al. [20]. Here, a non-associated plane stress anisotropic constitutive model with mixed isotropic/kinematic hardening is used. The quadratic Hill 1948 and non-quadratic Yld-2000-2d yield criteria of Barlat et al. [21] are considered in the non-associated flow rule model. In this work, the resulting problem is reduced to 10 equations. Solving these 10 equations, at each integration point, need a large computation time. Taherizadeh et al. [22,23] developed an anisotropic material models based on both associated and non-associated flow rules, respectively and mixed isotropic/kinematic hardening. These models are implemented into a user-defined material (UMAT) subroutines for the commercial finite element code ABAQUS to predict the springback of Numisheet'05 Benchmark#3. In these two works, the consistent tangent matrix is not obtained by exact linearization of the algorithm but by only modifying the elastic tangent matrix in the continuum elastoplastic tangent matrix. Vladimirov et al. [24] discussed the application of a finite strain model to predict springback in sheet forming. Both isotropic and kinematic hardenings are used in a new algorithm based on the exponential map. The resulting problem is reduced to 13 equations.

The objective of this work is to develop a one-equation integration algorithm of a generalized quadratic yield criterion with consistent tangent operator based on the mixed non-linear isotropic/kinematic hardening models of Chaboche. The plastic multiplier appears as the only unknowns in the resulting problem which will be solved using the Newton–Raphson iterative scheme. The use of consistent tangent operator allows the iterations number reduction. The generalized quadratic yield model with isotropic/kinematic hardening includes the quadratics criterion of Hill and J_2 plasticity as a special case. The numerical treatment of the suggested model is implemented on ABAQUS/Standard using user interface material subroutines (UMAT).

This work is organized as follows. In Section 2, the constitutive equations for modeling Hill and J_2 plasticity with nonlinear isotropic/kinematic hardening are briefly summarized. In Sections 3 and 4, the continuum tangent modulus and the one single scalar equation are presented respectively. The consistency of the stress-computation Algorithm and the Newton iteration method is described in Section 5. A number of numerical simulations are presented in Section 6 and closing remarks are stated in Section 7.

2. Constitutive equations

The partition of the total strain into an elastic part and a plastic part is assumed as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p \tag{1}$$

where $\boldsymbol{\varepsilon}^e$ denotes the elastic strain tensor and $\boldsymbol{\varepsilon}^p$ is the plastic strain tensor. The Helmholtz free energy is given by

$$\psi(\boldsymbol{\varepsilon}^e, \kappa, \boldsymbol{\alpha}_k) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \boldsymbol{D} : \boldsymbol{\varepsilon}^e + \sum_{k=1}^{M} \frac{a_k}{2}\boldsymbol{\alpha}_k : \boldsymbol{\alpha}_k + \psi_{iso}(\kappa) \tag{2}$$

where $\boldsymbol{D}$ is the general elastic operator, a_k are material parameters, $\psi_{iso}(\kappa)$ is the isotropic hardening function and κ and $\boldsymbol{\alpha}_k$ model the isotropic and kinematic hardening respectively. The states laws are

$$\boldsymbol{\sigma} = \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}}, \quad \boldsymbol{X}_k = \frac{\partial \psi}{\partial \boldsymbol{\alpha}_k}, \quad R = \frac{\partial \psi}{\partial \kappa} \tag{3}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{X}_k$ are stress and back-stress tensor respectively and R is the drag stress in isotropic hardening. Eqs. (2) and (3) give the following set of equations:

$$\boldsymbol{\sigma} = D{:}\boldsymbol{\varepsilon}^e, \quad \boldsymbol{X}_k = a_k\,\boldsymbol{\alpha}_k, \quad R = \frac{\partial \psi_{iso}}{\partial \kappa} \tag{4}$$

The yield function, f, and plastic potential, F, are assumed to be

$$f = \sqrt{\frac{3}{2}}\varphi_f(\boldsymbol{\xi}) - (\sigma_Y + R) \le 0, \quad \boldsymbol{\xi} = \boldsymbol{\sigma} - \boldsymbol{X}, \quad \boldsymbol{X} = \sum_{k=1}^{M} \boldsymbol{X}_k \tag{5}$$

$$F = \sqrt{\frac{3}{2}}\varphi_F(\boldsymbol{\xi}) - (\sigma_Y + R) + \frac{1}{2}\sum_{k=1}^{M}\left(\frac{b_k}{a_k}\boldsymbol{X}_k : \boldsymbol{X}_k\right) \tag{6}$$

where b_k are material parameters, σ_Y is the initial yield stress, $\boldsymbol{\xi}$ is the effective stress tensor and $\varphi_f(\boldsymbol{\xi})$, $\varphi_F(\boldsymbol{\xi})$ are continuously differentiable functions called yield function and plastic potential function respectively. These functions can be isotropic or orthotropic, quadratic or non-quadratic. An associative generalized quadratic yield function will be considered

$$\varphi_f(\boldsymbol{\xi}) = \varphi_F(\boldsymbol{\xi}) = \varphi(\boldsymbol{\xi}), \quad \varphi(\boldsymbol{\xi}) = \sqrt{\boldsymbol{\xi}^t P \boldsymbol{\xi}} \tag{7}$$

where $\boldsymbol{P}$ is a fourth order tensor which define the yield criterion. This yield function includes the classical J_2 plasticity yield condition and the quadratic Hill criterion as special cases. The evolution laws are

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma}\frac{\partial F}{\partial \boldsymbol{\sigma}} = \dot{\gamma}\sqrt{\frac{3}{2}}\frac{\partial \varphi}{\partial \boldsymbol{\sigma}}, \quad \dot{\boldsymbol{\alpha}}_k = -\dot{\gamma}\frac{\partial F}{\partial \boldsymbol{X}_k} = \dot{\gamma}\left[\sqrt{\frac{3}{2}}\frac{\partial \varphi}{\partial \boldsymbol{\sigma}} - \frac{b_k}{a_k}\boldsymbol{X}_k\right], \quad \dot{\kappa} = -\dot{\gamma}\frac{\partial F}{\partial R} = \dot{\gamma} \tag{8}$$

where $\dot{\gamma}$ is the plastic multiplier. Eq. (4b) with (8b) give

$$\dot{\boldsymbol{X}}_k = a_k\dot{\boldsymbol{\alpha}}_k = a_k\dot{\boldsymbol{\varepsilon}}^p - b_k\dot{\gamma}\boldsymbol{X}_k \tag{9}$$

Finally, the loading/unloading conditions, formulated in standard Kuhn–Tucker form, are as follows:

$$f \le 0 \quad , \quad \dot{\gamma} \ge 0 \quad , \quad \dot{\gamma} f = 0 \tag{10}$$

For convenience, the main relations are summarized in Table 1.

Remark 1. In general, the isotropic hardening function $\psi_{iso}(\kappa)$ in Eq. (2) is defined by using power or exponential laws

$$\begin{cases} \psi_{iso} = \frac{k}{n+1}\kappa^{n+1} \\ R = k\kappa^n \end{cases} \text{or} \begin{cases} \psi_{iso} = Q\left(\kappa + \frac{e^{-\beta\kappa}}{\beta}\right) \\ R = Q\left(1 - e^{-\beta\kappa}\right) \end{cases} \tag{11}$$

where k, n, Q and β are material parameters.

Remark 2. Hill yield criterion, in three-dimensional cases, is obtained by taking

$$\boldsymbol{P} = \frac{2}{3}\boldsymbol{H}, [\boldsymbol{H}] = \begin{bmatrix} H+G & -H & -G & 0 & 0 & 0 \\ & H+F & -F & 0 & 0 & 0 \\ & & F+G & 0 & 0 & 0 \\ & & & 2N & 0 & 0 \\ & Sym & & & 2M & 0 \\ & & & & & 2L \end{bmatrix} \tag{12}$$