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Vibrations and stability analysis of multiple rectangular plates coupled with elastic layers based on different plate theories

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ABSTRACT

Analytical solutions of natural frequencies and critical buckling loads for a set of homogenous plates in the complex plate system are determined in this paper. A different layerwise approach is found via interconnecting several plates into one kind of layered block. For this determination, the classical, first order and high order shear deformation theories are used to present rotary inertia and shear effects on systems with different numbers of elastically connected plates. Closed form solutions are determined in the form of explicit expressions of natural frequencies and critical buckling loads. General mode shapes are presented for systems of elastically connected plates. The stability of the complex system is determined and discussed for different stiffnesses of the support, layers and various geometric dimensions. These results are of considerable practical interest and have a wide application in the engineering practice of frameworks. The range of frequencies depending on number of elastically connected plates in the block is discovered.

1. Introduction

Multiple plate structures systems occupy an important place in theoretical research. These systems are also of great practical importance in many fields of structural engineering, like aeronautical and mechanical engineering applications where they are used for designing lightweight structures. The need for better approximations of results is always present especially in the case of models that are more complex. Solutions obtained from theoretical research of this type of structures may be used in civil engineering for creating an approximate procedure used to determine the behavior of multi-story buildings. Better approximations of obtained solutions sometimes are connected with the effects which are taken into account. In this case, various plate theories can lead to differences in approximations of solutions and comparison study between them is always of great interest.

The simplest theory which describes plate vibration – classical plate theory (CPT), does not take into account rotary inertia and shear deformation of the plate, i.e. the plate is considered infinitely stiff in transverse shear. This theory only takes into account flexural deformations. Therefore, classical plate theory has satisfactory accuracy only for truly thin plates. When the ratio of thickness of a plate is relatively large ($h/L>0.1$), classical plate theory leads to a significant overprediction of natural frequencies and buckling loads of plates and underprediction of deflections and stresses. In order to obtain more accurate results related to plate vibrations, deflections and stresses, numerous first order shear deformation theories were developed. Mostly used among them is the Mindlin–Reissner theory of plates [1]. The problem with first order shear deformation theory (FSDT) is the fact that those theories take the transverse shear stress to be constant through the plate thickness. In reality, the distribution of transverse shear stresses is parabolic through the plate thickness, and has the value of zero on the top and the bottom surface of a plate. The correction of this inequality is done by inserting the shear correction factor into the equations. The selection of the value for the shear correction factor can have an influence on over prediction or under prediction of the value of natural frequencies. In order to improve the accuracy of shear deformation plate theories, higher order shear deformation theories were developed. The most well-known among them is high order shear deformation theory (HSDT) developed by Reddy and Phan [2]. In addition, Reddy and Wang [3] gave an overview of relationships between classical and shear deformation plate theories.

The vibration phenomenon of a double-plate rectangular system has been studied in numerous papers. Oniszczuk [4] studied the vibration of a rectangular double plate system using the classical plate theory. Zhang et al. [5] studied influences of axial forces on the vibration of a double beam system using the Euler beam theory. Stojanović and Kozić [6] considered influences of axial forces, rotary inertia and shear on the vibration of a double beam system. Kukla [7] analyzed free vibrations of two elastically connected plates using a Green's function. In analytical research of vibrations of linear systems

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of plates it is usual to apply the Navier method. The Navier method [8–14], being a particular case of the Lévy method, is appropriated for analyzing a plate with all four edges simply supported. In this work, the Navier procedure is applied for formulating the complete exact theoretical solutions for the free transverse vibrations of a simply supported rectangular multiple plate system.

The importance of more accurate results of natural frequencies, amplitudes and buckling load, increases in the case of complex models, for example, the case of elastically connected plates. The present paper provides an analytical solution and results for further vibration analysis of systems of elastically connected thin and thick isotropic plates.

Some recent investigation in coupled plate systems, [15] where the free vibration of a bundle of identical rectangular plates fully in contact with an ideal liquid is analyzed and the effect of the liquid gap between the plates on the normalized natural frequencies is discussed, can be interesting for expanding the present study. Another recent research of Shyong and Chang [16] of the vibrations of a multi-step beam, where different sections of the beam have discreetly different characteristics, and exact equations which describe vibrations of the entire system are very useful, can be also very important for the future consideration of the present study. In the case of multiple plate system, there is an assembly of different parts, which interact between each other in some way. Finding a method, which can mathematically describe the behavior of the entire system like this, can be significant.

2. Theoretical formulation

The multiple plate system's vibration analysis has been done on the basis of the physical model of the vibrating system which is composed of multiple parallel rectangular plates connected by a Winkler elastic layer resting on an elastic foundation (see Fig. 1a and b). It is assumed that the plates are homogeneous and isotropic, and that they have constant thickness. Only the case of the plates having all edges simply supported is considered. In general, it is also assumed that the plates

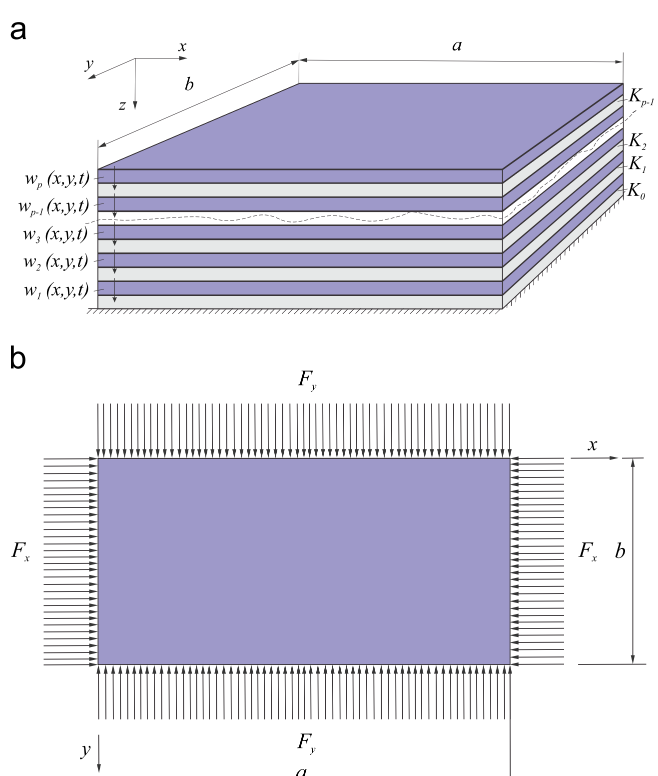

Fig. 1. (a) A set of p elastically connected plates on a Winkler type foundation and (b) uniformly compressed complex plate model (all plates are uniformly compressed).

are subjected to axial compression's distributed continuous loads. The small undamped vibrations of the system are analyzed. For determining the natural frequencies and critical buckling loads of multiple plate systems presented in this paper, trigonometric method previously employed in the paper by Stojanović et al. [17] is used, in which vibration analysis of multibeam system was described. The main problem in this kind of multiple plate system was an elimination of variables (rotations) when the first and high order theory is used because of implementation of the trigonometric method. Authors made a program for eliminating the variables given in the Appendix of the book of Stojanović and Kozić [17].

The plates have plan-form dimensions a and b with constant thickness h. All of the plates can be subjected to axial compressive loading in the direction of x axis and/or y axis. In the case of a rectangular multiple plate system, the transverse vibrations are described by the following differential equations obtained from the principle of virtual displacements and zero value of total virtual work, applied on the system of p plates in which each plate is loaded with the axial forces Fx and Fy. Analogously to the equations of motion with neglected rotary inertia and shear effects of the multibeam systems [19], when the classical plate theory is used for the multiple plate system, equations of motion have the following form:

$$D_1\ \nabla^4 w_1 + h_1\rho_1\frac{\partial^2 w_1}{\partial t^2} + F_x\frac{\partial^2 w_1}{\partial x^2} + F_y\frac{\partial^2 w_1}{\partial y^2} + K_0 w_1 - K_1(w_2 - w_1) = 0, \tag{1a}$$

$$D_i\ \nabla^4 w_i + h_i\rho_i\frac{\partial^2 w_i}{\partial t^2} + F_x\frac{\partial^2 w_i}{\partial x^2} + F_y\frac{\partial^2 w_i}{\partial y^2} + K_{i-1}(w_i - w_{i-1}) - K_i(w_{i+1} - w_i) = 0,\ i = 2,\ 3,\ \ldots, p-1 \tag{1b}$$

$$D_p\ \nabla^4 w_p + h_p\rho_p\frac{\partial^2 w_p}{\partial t^2} + F_x\frac{\partial^2 w_p}{\partial x^2} + F_y\frac{\partial^2 w_p}{\partial y^2} + K_{p-1}(w_p - w_{p-1}) = 0. \tag{1c}$$

where

$$D_i = \frac{E_i h_i^3}{12(1-\nu_i^2)}. \tag{2}$$

Eqs. (1a)–(1c) present a system of equations which describe transversal vibrations of a set of p elastically connected, identically axially loaded plates connected by elastic layers to a Winkler type foundation of the system presented in Fig. 1a, b.

Each plate is made of some material with Young's modulus E_i, mass density ρ_i, and uniform cross-sectional area A_i. All of the plates are subjected to the same uniformly compressive axial loading Fx and Fy. The first plate is connected with the ground by a Winkler foundation of the stiffness module K_0. The other plates are connected with each other also by a continuous linear elastic layer of Winkler type with stiffness module K_i. The transverse displacements of the plates are $w_i = w_i(x, y, t)$.

The transverse vibrations of a rectangular multiplate system using first order shear deformation theory are described by the following differential equations obtained from the principle of virtual displacements and zero value of total virtual work, applied on the system of p plates

first plate

$$\rho h\frac{\partial^2 w_1}{\partial t^2} - kGh\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial \psi_{x1}}{\partial x} + \frac{\partial \psi_{y1}}{\partial y}\right) + F_x\frac{\partial^2 w_1}{\partial x^2} + F_y\frac{\partial^2 w_1}{\partial y^2} + K_0 w_1 - K_1(w_2 - w_1) = 0, \tag{3a}$$

$$\frac{D}{2}\left[(1-\nu)\left(\frac{\partial^2 \psi_{x1}}{\partial x^2} + \frac{\partial^2 \psi_{x1}}{\partial y^2}\right) + (1+\nu)\frac{\partial}{\partial x}\left(\frac{\partial \psi_{x1}}{\partial x} + \frac{\partial \psi_{y1}}{\partial y}\right)\right] - kGh\left(\frac{\partial w_1}{\partial x} + \psi_{x1}\right) - \frac{\rho h^3}{12}\frac{\partial^2 \psi_{x1}}{\partial t^2} = 0, \tag{3b}$$