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On mixed isotropic-distortional hardening

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ABSTRACT

Mixed isotropic-distortional hardening allows for individual stress–plastic strain relations in different straining directions. Such hardening can be obtained by allowing the parameters in the effective stress function depend on anisotropy functions of the equivalent plastic strain. A methodology to calibrate these anisotropy functions is proposed in this work, and is demonstrated on an austenitic strainless steel. A high exponent eight parameter effective stress function for plane stress states is utilised. The anisotropy functions are calibrated by the use of experimental data from uniaxial tensile test data in three material directions and a balanced biaxial test. The plastic anisotropy is evaluated at a finite number of plastic strains, and it is assumed to vary piecewise linearly with respect to the equivalent plastic strain. At each level of plastic strain, the anisotropy is correctly represented, even if rather large increments in plastic strain are used in the calibration. It was found that there are at least two sets of anisotropy functions which satisfy the conditions in the calibration procedure. The resulting uniaxial stress–strain relations from the two sets of anisotropy functions in four additional straining directions, not included in the calibration set, were compared to the corresponding experimental data. From this validation, one of the anisotropy function sets could be discarded, whereas the other one gave a good prediction of the stress–strain relations in all the four additional directions.

1. Introduction

The demand for lighter and stronger products has increased the use of high strength steels. In general, high strength steels, such as martensitic steels, have a high initial yield strength but suffer from a less ductility, which results in less formability. Thus, the choice of material in a sheet metal application is often a compromise between strength and formability. In order to fully utilise the formability of the material, there is a need for accurate formability predictions. One major material failure mode which limits the formability is plastic instability, as this causes strain localisation and eventually a subsequent material fracture. In order to accurately predict this instability, the plastic hardening and anisotropy of the material are important.

Plastic anisotropy in metals has traditionally been described by anisotropic effective stress functions, giving anisotropic yield surfaces. Recent development has resulted in sophisticated multi-parameter functions [1,2], which are able to account both for anisotropy in the yield stresses and in the plastic flow. The response under proportional loading is generally governed by the shape of the yield surface and the plastic hardening function. However, it has been shown that both a translation and a distortion of the yield surface take place during plastic deformation, see [3]. Thus, kinematic and distortional hardening can be mixed in order to describe the material response under non-proportional loadings [3,4]. A similar effect can be achieved with a mixed isotropic-kinematic hardening since kinematic hardening is a special case of anisotropic hardening [5]. The yield stress changes differently at other stress states than the state defined by the current loading. The introduction of a back stress tensor, such as is used in the kinematic hardening approach, removes the homogeneity of the effective stress function. A new approach was proposed in [6] for describing the Bauschinger effect, while keeping the homogeneity of the effective stress function by introducing an additional fluctuating tensor. By their approach, the resulting yield surface contains the origin and is thus unable to account for reversed plastic yielding while unloading along the loading path.

The calibration procedure of the *anisotropy parameters* is often based on a least square fit to a set of target experimental values. The validity of the resulting yield condition for stress states not included in the fit has usually been investigated by comparing predicted results to corresponding experimental data. For example, the variations of the uniaxial yield stress and R-value with angle to the rolling direction, RD, have been investigated in [7].

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The yield surface is usually calibrated to experimental data at the onset of plastic deformation. If isotropic or mixed isotropic-kinematic hardening is used, the plastic hardening under proportional loading becomes similar regardless of stress state. Even though the initial plastic anisotropy may be well described by the yield surface, this approach is insufficient to describe different hardening rates in different directions. However, such anisotropic hardening has been considered as important for formability predictions, see [8,9]. Plunkett et al. [10] described the varying anisotropy by letting the anisotropy parameters of the effective stress function depend on the equivalent plastic strain. Such varying anisotropy parameters governing the plastic anisotropy are henceforth denoted as *anisotropy functions*. Using this approach, not only the initial plastic anisotropy, but also the subsequent anisotropic plastic hardening, is represented. Aretz [9] used a similar approach with the eight parameter effective stress function YLD2003 [11], in which the anisotropy functions varied with plastic work, and showed that mixed isotropic-distortional hardening is important for predicting the anisotropy in forming processes. The hardening approach presented in this paper gives a similar response as ordinary isotropic hardening in the case of non-proportional hardening, however, a combination with kinematic hardening is straightforward.

Mixed isotropic-distortional hardening is further examined herein. The eight parameter effective stress function YLD2003 has been used. YLD2003 is defined by eight anisotropy parameters and requires at least eight experimental data points for calibration. The anisotropy functions were calibrated to the evolution of four yield stress ratios and four plastic strain ratios. In this work, the anisotropy functions were assumed to be piecewise linear, i.e. the anisotropy functions were calibrated at a finite number of equivalent plastic strain levels. If the functions are calibrated at a small number of plastic strains, this will affect the result. It was shown that the anisotropy functions depend not only on the anisotropy but also on the increment in equivalent plastic strain between the vertices. The anisotropy functions are calibrated by a novel calibration method, which takes the piecewise linear variation of the anisotropy functions into account. With this method, only a small number of calibrations have to be conducted in order to define the mixed isotropic-distortional hardening. The method is demonstrated on an austenitic stainless steel, HyTens 1000, within the EN 1.4310 standard. Moreover, it was found that for a set of eight experimental target values describing the plastic anisotropy, two solutions to the anisotropy functions exist, both of which describe the target values exactly. Therefore, to find the unique solution, uniaxial tensile tests in four additional material directions and shear tests in two material directions were carried out.

In Section 2, the experimental procedures and data used in the calibration are presented. In the subsequent sections, the constitutive model along with the calibration procedure is described. It was found that two alternative sets of anisotropy functions fulfill the target values in the calibration procedure of the anisotropy functions. In Section 6, the predicted uniaxial stress–strain relations in four additional directions, which not are included in the calibration procedure, are compared to the corresponding experiments in order to test the validity of the two solutions. Furthermore, the force–displacement relations in two shear tests obtained from the two solutions were compared to the corresponding experimental results in order to further distinguish between the two solutions.

Table 1
Chemical composition of the investigated steel grade, HyTensX (%).

C	Si	Mn	P	S	Cr	Ni	N
0.094	0.72	1.11	0.022	0.001	16.9	7.1	0.021

2. Experimental work

2.1. Material

The material under investigation is an austenitic stainless steel of the brand HyTensX, see [13], within the EN 1.4310/AISI 301 standard. The material is cold rolled, and the resulting steel grade is denoted as HyTens 1000 or 1.4310 C1000. Its chemical composition is presented in Table 1. The thickness of the steel is 1.5 mm. HyTensX is meta-stable, i.e. phase transformation takes place during deformation. The authors have previously investigated strain ageing in this material, see [14]. It was found that the material is sensitive to both dynamic and static strain ageing. The phase transformation model proposed by [15], combined with a linear mixture rule of the yield stresses in the two phases, was used in order to account for martensitic transformation. Recently, the martensitic transformation in the same material (HyTensX) was investigated in [16] using an enhanced macroscopic model, in which the effect of the triaxiality on phase transformation was included.

2.2. Tensile tests

Uniaxial tensile tests were conducted in the directions $\phi = 0, 15^\circ$, 30°, 45°, 60°, 75° and 90° with respect to the RD . The geometry of the test specimen is shown in Fig. 1. A mechanical extenso-meter with initial length $l_0 = 25$ mm was attached to the specimen in order to measure the longitudinal strain, $\varepsilon_l = \ln(l/l_0)$. A second extenso-meter was attached in order to measure the width change, $w - w_0$, in order to evaluate the transversal strain $\varepsilon_w = \ln(w/w_0)$. The deformation was controlled by applying a constant crosshead velocity $v = 3$ mm/min, resulting in an approximate overall strain rate of $\dot{\varepsilon}_l \approx 10^{-3}$ s^{-1}. Furthermore, the force, F, was continuously measured in order to evaluate the stress, $\sigma = F \exp(\varepsilon_l)/A_0$, where A_0 is the undeformed cross section area. It can be noticed that the local strain rates were significantly higher due to serrated yielding, something which is not considered in the present work.

Three repetitions were conducted in each direction. The specimen to specimen variation of the stress–strain relations was found to be small. In fact, it may be noticed that the stairs in the stress–

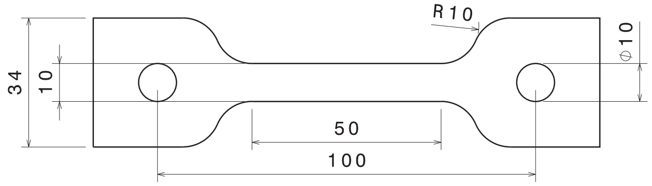

Fig. 1. Geometry of the tensile specimen.

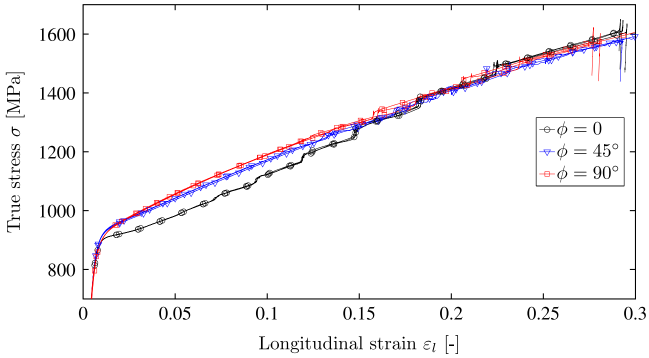

Fig. 2. True stress–longitudinal strain relations from tensile tests. Three tests were conducted in each material direction.