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An analytical model for the face wrinkling failure prediction of metallic corrugated core sandwich columns in dynamic compression

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ABSTRACT

This work was motivated by a necessity of simple and efficient models approximating the responses associated with all the possible failure modes of corrugated core sandwich columns in dynamic compression. In this study, an analytical model was proposed for the prediction of face wrinkling behavior of corrugated core sandwich columns under dynamic compressive loading *perpendicular-to-corrugations*. The proposed analytical model has been based on the calculation of transverse motion of face ligaments near the front and back ends of a sandwich column. The governing equation for the face ligaments has been a dynamic version of Euler–Bernoulli beam–column equation, and the motions were obtained by employing the Galerkin method to solve the equation. For validation of the proposed model, FE simulations were performed to compare several metrics such as reaction force and the transverse deflection of face ligaments. It was revealed that the dynamic face wrinkling response was also affected by overall column length and the rate of loading unlike the face wrinkling strength of corrugated core sandwich columns in quasi-static compression. Finally, the model's limitation and efficiency were discussed.

1. Introduction

Many studies demonstrated that the metallic core sandwich structures with periodic cellular metals (PCM) have many advantages over the other structural elements at the structural point of view [1–3]. In addition, a large body of a literature reported that the structures were able to enjoy their multifunctional benefits by taking advantage of their porosity: heat exchanger, energy absorption, flow channel etc. [4–6]. Prior to making use of these attractive features, their mechanical response should be investigated against various external loads, which could possibly happen in real-life structures.

Periodic cellular metals can be potentially applicable in the dynamic loading environment. Compared to the rich and long history of quasi-static studies [7–14], dynamic studies have only recently started to receive attention. For the last decade, the dynamic out-of-plane response of metallic core sandwich structures has been actively studied for blast mitigation [15–22]. Their studies investigated not only the deformation and momentum transfer mechanisms subject to blast loading but also the dynamic performance depending on their design variables, which include core and parent material selections.

On the other hand, the dynamic in-plane response of sandwich columns with PCM cores has not received as much interest as the dynamic out-of-plane response does. To design mechanical components and structures using periodic cellular metals properly, the relationship between various sandwich design variables and their dynamic response must be studied [23]. In addition, the dominant failure mechanism under dynamic effects should be identified depending on the design variables since sandwich structures employing PCM fail in many ways. For example, the failures of corrugated core sandwich columns are largely divided into global buckling and face wrinkling competing with each other [2]. In quasi-static compression, the strengths resisting against global buckling and face wrinkling are simply given as functions of their design variables and material properties, $P_{QS}^{(GB)}(\mathrm{DVs}, \mathrm{Material})$ and $P_{QS}^{(FW)}(\mathrm{DVs}, \mathrm{Material})$. If the columns are designed such that $P_{QS}^{(GB)} > P_{QS}^{(FW)}$, they fail by face-wrinkling, and vice versa. Consequently, it is essential to understand each failure mode depending on design variables, especially for sandwich structures employing PCMs. On the other hand, it is not so easy to predict individual response under dynamic loading due to complex dynamic effects. Furthermore, the competitions between the failure modes quite differ when dynamic effects are involved [24]. For this reason, we need an analytical tool approximating the

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dynamic response associated with individual failure response under dynamic circumstances for proper design. Recently, the authors studied the dynamic global buckling response of corrugated core sandwich structures and presented a semi-analytical model approximating the response [25]. But, local failure mechanisms cannot be ignored at the design consideration of metallic corrugated core sandwich structures.

In this study, an analytical model will be proposed to predict dynamic face wrinkling failure of corrugated core sandwich columns compressed perpendicular-to-corrugations. This paper is outlined as follows. Section 2 describes the problem of metallic corrugated core sandwich columns compressed at increased rates and its mathematical formulation. Section 3 details the analytical model to solve the formulation given in Section 2. Section 4 presents the validation of the proposed model by comparison with the Finite Element simulations through some examples, and discusses its efficiency and limitations.

2. Problem statements

2.1. Problem definition

The response of SS304 and Al6061-T6 corrugated core sandwich columns dynamically loaded *perpendicular-to-corrugations* is studied as shown in Fig. 1. One end (front end) is axially displaced at a constant rate of V with all the other degrees of freedoms (DOFs) constrained while a fixed end condition is imposed at the other end (back end). The considered range of the compression rate, V, is such that the incident stress wave at the moment of imposing compression is purely elastic, $V/\left(c_{el}^{(perp)}\varepsilon_Y\right) < 1$. And thus, the maximum applied velocity satisfying the condition can be order of a few m/s for a typical design of corrugated core sandwich columns. Note that the apparent elastic stress wave speed, $c_{el}^{(perp)}$, is related to both material properties and sandwich geometric parameters, given as follows [23, 25]:

$$c_{el}^{(perp)} = \sqrt{\frac{E}{(1-\nu^2)}\frac{2h}{(2h+\bar{\rho}c)\rho}} \tag{1}$$

Material properties, E, ν, ρ denote Young's modulus, Poisson's ratio and parent material density and sandwich geometric parameters, h, c, and $\bar{\rho}$ are face thickness, core height and the core relative density, respectively. As considered in the global buckling study [25], the parent materials, SS304 and Al6061-T6, are modeled as a rate-dependent elastic–plastic material with bilinear strain-hardening (E=230769.2 MPa,$E_P^{(1)} = (d\sigma_{eq}/dp) = 8117.65$ MPa, $E_P^{(2)} = (d\sigma_{eq}/dp) =$ 2460.66 MPa, $\sigma_Y^{(0)} =$ (MPa), $\sigma_Y^{(1)} =$ (MPa)), D=4920, m=0.154) and a rate-independent elastic–plastic material with a linear strain hardening (E=75,150 MPa, σ_Y=293.9 MPa, $E_P = (d\sigma_{eq}/\ dp) = 534.2$ MPa). The rate dependence of SS304 is represented as follows:

$$R = \frac{\sigma_y(\dot{p})}{\sigma_y^0} = 1 + \left(\frac{\dot{p}}{D}\right)^m \tag{2}$$

2.2. Mathematical formulation

The proposed analytical model for the dynamic face wrinkling problem considers the behavior of a face sheet ligament between nodes as shown in Fig. 2. When one end of a corrugated core sandwich column is compressed in this loading configuration (*perpendicular-to-corrugations*), it is assumed that only the faces, not the corrugated core, resist the in-plane load because the main deformation mechanism of the core is not by stretching but by folding at nodes [23].

A face sheet ligament of thickness, h, and span length, $2\ell \cos\omega$, is modeled as an Euler–Bernoulli beam–column subject to a time-varying axial force, $P(t)/2$. The governing equation of motion based on the classical Euler–Bernoulli beam theory [26] for the face sheet ligament can be given by

$$D^{(f)}\frac{\partial^4 w^{(f)}}{\partial x^4} + \frac{\partial}{\partial x}\left(\frac{P(t)}{2}\frac{\partial w^{(f)}}{\partial x}\right) + \rho h\frac{\partial^2 w^{(f)}}{\partial t^2} = -\frac{\partial}{\partial x}\left(\frac{P(t)}{2}\frac{\partial w_0^{(f)}}{\partial x}\right) \tag{3}$$

The governing equation of motion (3) is given in terms of the flexural rigidity of a face sheet ligament, $D^{(f)}$, initial geometric imperfections of the face sheet ligament, $w_0^{(f)}$, axial force, $P(t)/2$, and density, ρ. The superscript (f) stands for a face sheet ligament between nodes. The flexural rigidity, $D^{(f)}$, is given by

$$D^{(f)} = \frac{Eh^3}{12(1-\nu^2)}, \text{ if the face material is elastic,} \tag{4a}$$

$$D^{(f)} = \frac{E_T' h^3}{12} \text{ if the face material is plastic.} \tag{4b}$$

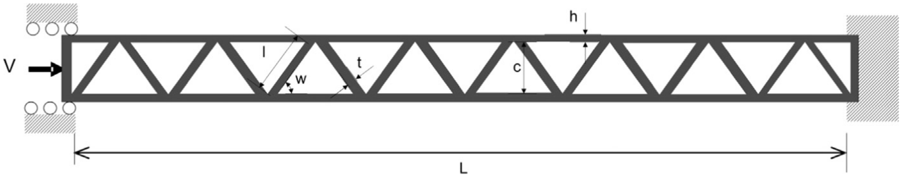

Fig. 1. Dynamic problem of face wrinkling failure.

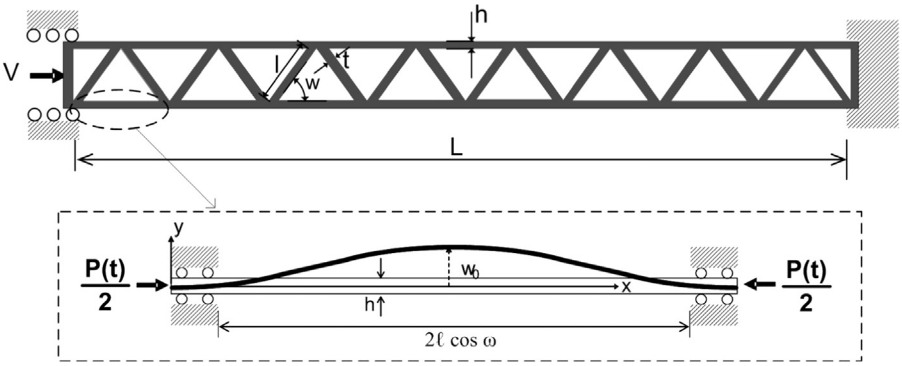

Fig. 2. Euler beam–column modeling of a face ligament for the dynamic face wrinkling.