International Journal of Mechanical Sciences 87 (2014) 1–17

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

On the frictional sliding contact problem between a rigid circular conducting punch and a magneto-electro-elastic half-plane

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ARTICLE INFO

Article history:
Received 11 October 2013
Received in revised form
17 March 2014
Accepted 24 April 2014
Available online 2 June 2014

Keywords:
Frictional sliding contact
Homogeneous magneto-electro-elastic material
Perfectly conducting circular punch
Closed-form solution
Singular integral equations

ABSTRACT

The frictional sliding contact problem between a homogeneous magneto-electro-elastic material (MEEM) and a perfectly conducting rigid circular punch subjected to magneto-electro-mechanical loads is considered in this paper. The problem is formulated under plane strain conditions and the resulting plane magneto-electro-elasticity equations are converted analytically using Fourier transform into three coupled singular integral equations in which the main unknowns are the normal contact stress, the electric displacement and the magnetic induction. The main contribution of this work is the derivation of an analytical closed-form solution for the normal contact stresses, electric displacement and magnetic induction distributions. The primary purpose of this investigation is to study the effect of the friction coefficient, the punch radius, the applied magneto-electro-mechanical loadings, the material composition on the contact surface and in-plane surface stresses, electric displacement and magnetic induction distributions for the case of a circular stamp profile.

1. Introduction

Recently, considerable effort has been deployed on magneto-electro-elastic materials (MEEMs), which are characterized by coupling among the elastic, electric, and magnetic fields. These materials include piezoelectric (PE), piezomagnetic (PM), magnetoelectric (ME) and magneto-electro-elastic (MEE) couplings [1–6]. These materials can exchange magneto-electro-elastic energy from one form to the other due to the coupling effect. Because of their advantages, several engineering applications have been found including electronic packaging, hydrophones, magnetic field probes, medical ultrasonic imaging and in general as transducers, sensors and actuators.

Devices made of piezoelectric materials involve contact between components of varying stiffness. This has motivated several researchers to study contact mechanics of piezoelectric materials. Authors generally reduced the three-dimensional problems involving coupling between mechanical, electric and thermal fields to two-dimensional ones using plane or axisymmetric assumptions. Matysiak [7] considered the contact problem between a rigid conducting punch and a piezoelectroelastic half-plane. Fan et al. [8] studied the stress and electrical field distributions in a piezoelectric half-plane under a contact load using Stroh's formalism. The thermal contact problem of a piezoelectric layer under a sliding flat punch is investigated by Zhou and Lee [9]. A complete general theory for the axisymmetric contact problem of piezoelectric solids is proposed by Ginnakopoulos and Suresh [10]. The contact response of an arbitrarily multilayered piezoelectric half-plane indented by a rigid frictionless parabolic punch is considered by Guillermo and Paul [11] based on stiffness matrix formulation. The axisymmetric contact problem between an insulating or conducting circular punch and a piezoelectric layer or half-plane is considered by Wang and Han [12].

In addition, contact mechanics of piezomagnetic materials was considered to characterize their coupling properties. The effects of the moving punch speed on the stress and magnetic induction were investigated by Zhou and Lee [13]. They considered the frictional sliding contact problem between a rigid punch and a piezomagnetic half plane. Ginnakopoulos and Parmaklis [14] solved the axisymmetric problem of piezomagnetic solids under specific magnetic boundary conditions and concluded that the coupling between the elastic and magnetic fields would lead to a significant effect on the indentation force.

However, the contact problem of coupled magneto-electro-elastic materials involves additional coupling which render the

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http://dx.doi.org/10.1016/j.ijmecsci.2014.04.024
0020-7403/

contact problem more complex. An exact solution for the elliptical Hertzian contact of a transversely isotropic magneto-electro-elastic media for both smooth and frictional contact cases is developed by Hou et al. [15]. Chen et al. [16] studied the frictionless axisymmetric contact problem between a rigid punch and a magneto-electro-elastic half-space by considering various types of electric and magnetic boundary conditions. Rogowski and Kalinski [17] considered the axisymmetric frictionless contact problem of magneto-electro-elastic half-plane indented by a truncated conical punch assumed to be perfect electric and magnetic conductor. Elloumi et al. [18] obtained closed-form solutions for the frictional sliding contact problem of a magneto-electro-elastic half-plane indented by a flat punch. Zhou and Kim [19] studied the frictional sliding contact problem of magneto-electro-elastic materials under a perfectly insulating rigid punch of a triangular or circular profile.

To the best of our knowledge, the problem involving at the same time a frictional contact and a perfectly conducting circular punch for the case of a homogeneous magneto-electro-elastic half-plane has not been solved in the published literature to-date. The motivation of this work is to study the sliding contact problem between a homogeneous transversely isotropic magneto-electro-elastic half-plane and a rigid circular punch subjected to a normal load P, a tangential load Q, an electric charge T and a magnetic charge J. The mixed boundary value problem is solved analytically using Fourier transform to convert the plane magneto-electro-elasticity equations into three coupled singular integral equations. An analytical closed-form solution is obtained for the normal contact stresses, electric displacement and magnetic induction distributions.

This paper is organized as follows. The problem description and formulation is given in Section 2. The problem solution is detailed in Section 3. The analytical solution of the problem is presented in Section 4. The numerical results are then discussed in Section 5. Finally, concluding remarks are given in Section 6.

2. Problem description and formulation

The plane strain contact problem under consideration is described in Fig. 1. The stiff contacting element is represented by a rigid circular punch subjected to the action of a normal load P and a tangential load Q. It is assumed that the punch is a perfect electro-magnetic conductor with constant electric and magnetic potentials in response to the indentation of an electric charge T and a magnetic charge J. The half-plane is represented by a homogeneous transversely isotropic magneto-electro-elastic half-plane with polarization in the y-direction. It is assumed that the

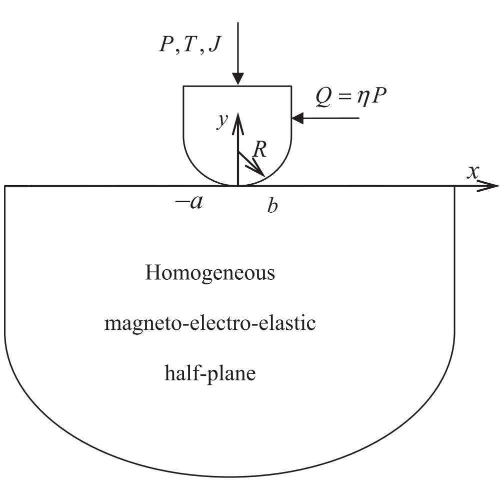

Fig. 1. Geometry and loading of the sliding contact problem for a circular punch.

stamp and the half-plane are in relative motion and the friction coefficient η, along the contact region is constant (i.e. $Q=\eta P$).

In Cartesian coordinates (x,y), the linear constitutive equations of a transversely isotropic magneto-electro-elastic material are given by Chen et al. [20]

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}=\begin{bmatrix}c_{110} & c_{130} & 0\\ c_{130} & c_{330} & 0\\ 0 & 0 & c_{440}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xy}\end{Bmatrix}-\begin{bmatrix}0 & e_{310}\\ 0 & e_{330}\\ e_{150} & 0\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\end{Bmatrix}-\begin{bmatrix}0 & f_{310}\\ 0 & f_{330}\\ f_{150} & 0\end{bmatrix}\begin{Bmatrix}H_x\\ H_y\end{Bmatrix}, \tag{1a}$$

$$\begin{Bmatrix}D_x\\ D_y\end{Bmatrix}=\begin{bmatrix}0 & 0 & e_{150}\\ e_{310} & e_{330} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xy}\end{Bmatrix}+\begin{bmatrix}\varepsilon_{110} & 0\\ 0 & \varepsilon_{330}\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\end{Bmatrix}+\begin{bmatrix}g_{110} & 0\\ 0 & g_{330}\end{bmatrix}\begin{Bmatrix}H_x\\ H_y\end{Bmatrix}, \tag{1b}$$

$$\begin{Bmatrix}B_x\\ B_y\end{Bmatrix}=\begin{bmatrix}0 & 0 & f_{150}\\ f_{310} & f_{330} & 0\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ 2\varepsilon_{xy}\end{Bmatrix}+\begin{bmatrix}g_{110} & 0\\ 0 & g_{330}\end{bmatrix}\begin{Bmatrix}E_x\\ E_y\end{Bmatrix}+\begin{bmatrix}\mu_{110} & 0\\ 0 & \mu_{330}\end{bmatrix}\begin{Bmatrix}H_x\\ H_y\end{Bmatrix}. \tag{1c}$$

where σ_{xx}, σ_{yy} and σ_{xy} are the components of the stress tensor, D_x and D_y are the components of the electric displacement along x and y, respectively, B_x and B_y are the components of the magnetic induction along x and y, respectively, ε_{xx}, ε_{yy} and ε_{xy} are the components of the strain tensor, E_x and E_y are the electric field components along x and y, respectively, H_x and H_y are the magnetic field components along x and y, respectively; c_{ij0}, e_{ij0}, f_{ij0}, g_{ij0}, $(i=1,3,4)$, $(j=1,3,4,5)$, are the elastic, piezoelectric, piezomagnetic and electromagnetic constants, respectively. ε_{ij0}, $(i,j=1,3)$, are the dielectric permittivities and μ_{ij0}, $(i,j=1,3)$, are the magnetic permeabilities.

The strain–displacement relationships, the electric field-electric potential relationships and the magnetic field-magnetic potential relationships, can be written as follows:

$$\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\quad \varepsilon_{xx}=\frac{\partial u}{\partial x},\quad \varepsilon_{yy}=\frac{\partial v}{\partial y}, \tag{2a–c}$$

$$E_x=-\frac{\partial \phi}{\partial x},\quad E_y=-\frac{\partial \phi}{\partial y},\quad H_x=-\frac{\partial \psi}{\partial x},\quad H_y=-\frac{\partial \psi}{\partial y}, \tag{2d–g}$$

where u and v are the displacements along x and y, respectively, and ϕ and $\psi$are the electric and magnetic potentials, respectively.

Neglecting body forces, the basic equations of the plane contact problem for homogeneous transversely isotropic magneto-electro-elastic solids are the equilibrium equations, Gauss law for the electric field and Gauss law for the magnetic field

$$\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{xy}}{\partial y}=0,\quad \frac{\partial \sigma_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}=0, \tag{3a, b}$$

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}=0,\quad \frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}=0. \tag{3c, d}$$

Substituting (2a)–(2g) into (1a), (1b) and (1c) and the resulting expressions into (3a)–(3d) yields the plane magneto-electro-elasticity partial differential equations in terms of displacements u and v, electric potential ϕ and magnetic potential ψ as the dependent unknowns

$$c_{110}\frac{\partial^2 u}{\partial x^2}+c_{440}\frac{\partial^2 u}{\partial y^2}+(c_{130}+c_{440})\frac{\partial^2 v}{\partial x \partial y}+(e_{310}+e_{150})\frac{\partial^2 \phi}{\partial x \partial y}$$