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Steady-state response of thermoelastic half-plane with voids subjected to a surface harmonic force and a thermal source

Y.Y. Zhu [a,*], Y. Li [b], C.J. Cheng [b]

[a] Department of Computer Science and Technology, Shanghai Normal University, Shanghai 200234, China
[b] Department of Mechanics, Shanghai University, Shanghai 200244, China

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ABSTRACT

In this paper, the steady-state response for a thermoelastic half-plane with voids is studied, in which, the surface of the half-plane is partly subjected to a surface harmonic force and a thermal source. The semi-analytical solutions and the numerical solutions of the half-plane problems with three different materials are obtained from a semi-analytical method and a developed differential quadrature element method in this paper, respectively. The corresponding numerical results are compared and the effects of parameters are considered as well. It can be seen that the semi-analytical solutions and corresponding numerical solutions coincide with each other. This means that the differential quadrature element method is a very efficient method for seeking the numerical solutions of the half-plane problems with discontinuity, and it has some advantageous properties, such as small computational amount, high accuracy, and better convergence.

1. Introduction

There exist abundant porous materials in nature, such as wood, sandy soil, sponge and coral. Also a lot of engineering materials are all porous materials in the natural state, such as concrete, wood, stone, and ceramic. Porous materials have extensive applications in aerospace, electronic communications, construction, metallurgy, nuclear energy, petrochemical industry, machine, medicine and environmental protection due to their advantageous properties, such as low relative density, light weight, high specific strength and surface area, thermal and acoustical insulation as well as good permeability. Hence, the application field of porous materials has already outdistanced homogenous materials.

Since the 1980s, many researchers have been interested in studying the theories and applications for thermoelastic materials with voids. A series of relative perfect theories for thermoelastic materials with voids has been presented by Goodman and Cowin [1], Nunziato and Cowin [2], Cowin and Nunziato [3], Iesan [4], and also many recent applied results have been achieved. Scarpetta [5] presented the minimum principle for the bending problem of elastic plates with voids. Bîrsan [6] gave a bending theory of porous thermoelastic plates, and then, Bîrsan [7,8] presented a nonlinear theory for porous elastic and thermoelastic shells and discussed the corresponding linearized theory. Sharma et al. [9] analyzed the three-dimensional vibration of a thermoelastic cylindrical panel with voids. Sharma and Kaur [10] investigated the propagation of thermoelastic waves along the circumferential direction in homogeneous, isotropic, cylindrical curved solid plates with voids. Ghiba [11] presented the bending theory of Mindlin type thermoelastic plates with voids and studied the temporal behavior of the corresponding initial–boundary value problem. Ciarletta et al. [12] studied plane waves and vibrations of materials with voids from the theory of micropolar thermoelasticity. Chiriţă and Ciarletta [13] studied the structural stability for a mathematical model of the linear thermoelastic materials with voids. Li and Cheng [14] presented a Hamilton variational principle of anisotropic thermoelastic materials with voids and gave a relevant nonlinear model under the finite deformation. Due to that the elastic half-space and half-plane problems have extensive applications in practice of the dynamic response analyses of the foundation and so on, the relevant research has been studied for a long time. Recently, Kumar and Rani [15–17] applied the transform methods to study a series of problems for the thermoelastic half-space with voids subjected to a surface force or a thermal load.

In this paper, the steady-state dynamic response for a thermoelastic half-plane with voids subjected to a surface harmonic force and a thermal source is studied. A semi-analytical method (SAM) and a new numerical method, namely, the developed differential

* Corresponding author. Tel.: +86 21 56380560.
E-mail address: yuanyuan_zhu@hotmail.com (Y.Y. Zhu).

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Nomenclature

SAM	semi-analytical method
DQEM	differential quadrature element method
TEVHP	thermoelastic material with voids
TEHP	thermoelastic material without voids
EVHP	elastic material with voids
SAS	semi-analytical solution
DQM	differential quadrature method
u_i	displacement vector
ε_{ij}	strain tensor
σ_{ij}	stress tensor
ϕ	change in the volume fraction of voids
θ	change in the temperature fields
λ, μ	Lamé coefficients
ρ	density
χ	equilibrated inertia
α_v	diffusion coefficient of porosity variation
b_v	stress parameter of porosity variation
ξ_v	non-conservative characteristic coefficient of porosity variation
α	linear expansion coefficient
K	thermal conductivity
c_e	specific heat at the constant strain
m_v	void–heat coupling coefficient
T_0	absolute temperature at the natural state
δ_{ij}	Kronecker symbol
$\hbar$	heat transfer coefficient
$p(x,t)$	surface harmonic force
$Q(x,t)$	surface thermal source
Ω	circular frequency of excitation
X, Z	dimensionless forms of coordinates x, z
τ	dimensionless form of time t
U, W	dimensionless forms of displacements u, w
ψ	dimensionless form of ϕ
Θ	dimensionless form of θ
$\sigma_{ZZ}, \tau_{ZX}, \sigma_{XX}$	dimensionless forms of stresses
$\overline{P}$	dimensionless form of surface harmonic force
$\overline{Q}$	dimensionless form of surface thermal source
ω	dimensionless form of excitation frequency Ω
$\overline{\hbar}$	dimensionless form of heat transfer coefficient $\hbar$
$a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, V_1, V_3$	dimensionless parameters
$\overline{a}_0$	width of local area of the subjected surface force and/or thermal source
n_1, n_2	number of grid points collocated in the elements 1 and 2 along the X-direction, respectively
n_3	number of grid points collocated along the Z-directions
$U_{ij}^{(l)}, W_{ij}^{(l)}, \psi_{ij}^{(l)}, \Theta_{ij}^{(l)}$	values of the corresponding functions at the grid point (i,j) in the element l
$A_{ik}^{(m,l)}$	weighting coefficient of the mth order partial derivative with respect to X
$B_{jk}^{(n,l)}$	weighting coefficient of the nth order partial derivative with respect to Z

quadrature element method (DQEM), are applied to obtain the semi-analytical solution and numerical solution for the problem, respectively. One can see that the SAM has a unique function for solving the problems for the thermoelastic half-plane with voids due to that the analytical solution can be obtained in the transform field. At the same time, the comparison between the numerical solution and the corresponding semi-analytical solution points out that they are agreed well with each other. This means that the DQEM is a very efficient numerical method for seeking the numerical solutions of thermoelastic half-plane problems with discontinuity, and it has some advantages, such as small computational amount, high accuracy, and better convergence.

2. Basic equations and the formulation of the problem

If the body forces, thermal sources and external equilibrated body forces can be ignored, following the linear theory of thermoelastic materials with voids [4], we have the basic equations for a linear isotropic thermoelastic body with voids as follows:

$$\begin{cases} \mu u_{i,kk}+(\lambda+\mu)u_{k,ki}+b_v\phi_{,i}-\beta\theta_{,i}=\rho\ddot{u} \\ \alpha_v\phi_{,ii}-b_vu_{k,k}-\xi_v\phi+m_v\theta=\rho\chi\ddot{\phi} \\ K\theta_{,ii}-\beta T_0\dot{u}_{k,k}-m_vT_0\dot{\phi}=\rho c_e\dot{\theta} \end{cases} \quad (1)$$

where the first equation in Eq. (1) is the differential equation of motion, the second one is the evolution equation of voids, and the third one is the energy equation. At the same time, the constitutive equation is given as

$$\sigma_{ij}=\lambda u_{k,k}\delta_{ij}+\mu(u_{i,j}+u_{j,i})+(b_v\phi-\beta\theta)\delta_{ij} \quad (2)$$

In these equations, u_i, ε_{ij}, σ_{ij} are the displacement vector, strain tensor and stress tensor, respectively, and ϕ, θ are the changes in the volume fraction of voids and the temperature fields, respectively (hereafter, they are called as the volume fraction of voids and the temperature for short). The meaning of parameters, which are depending on the properties of materials, can be found in [4]. For example, λ, μ are the Lamé coefficients, ρ is the density in the reference configuration, χ is the equilibrated inertia, α_v, b_v, ξ_v are the void parameters which describe the characteristics of porosity variation. Exactly, α_v is a diffusion coefficient of porosity variation which determines the velocity of expansion wave in incompressible granular materials, b_v is a stress parameter of porosity variation, ξ_v is a non-conservative characteristic coefficient of porosity variation. $\beta=(3\lambda+2\mu)\alpha$, in which, α is the linear expansion coefficient. In addition, K is the thermal conductivity, c_e is the specific heat at the constant strain, m_v is a void–heat coupling coefficient, T_0 is the absolute temperature at the natural state and δ_{ij} is the Kronecker symbol.

From Eq. (1), it can be seen that if letting α_v, b_v, ξ_v, m_ν be zero, the system is reduced to one of the thermoelastic problems, and if letting m_ν, β, K be zero, the system is reduced to one of the elastic problems with voids, further, if letting α_v, b_ν, ξ_v, m_ν, β, K be zero, the system is reduced to one of the elastic problems.

For a 2-D generalized plane problem, it can be regarded as a plane stain problem. Thus, the displacement along the y-direction can be approximately regarded as zero, and all unknown quantities are independent of the coordinate y, namely, $u=u(x,z,t)$, $v=0$, $w=w(x,z,t)$, $\theta=\theta(x,z,t)$, $\phi=\phi(x,z,t)$. At this time, the material parameters λ, μ, b_v, β in Eq. (1) should be converted into $\lambda'=(2\lambda\mu/2\mu-\lambda)$, $\mu'=\mu$, $b_v'=(2\mu/\lambda+2\mu)b_v$, $\beta'=(2\mu/\lambda+2\mu)\beta$ (hereafter, λ,μ,b_v,β are still used to represent $\lambda',\mu',b_v',\beta'$ for convenience).

For definiteness, assume that the surface of the half-plane is subjected to a harmonic load $p(x,t)$ or a harmonic thermal source