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# Grain statistics induced size effect in the expansion of metallic micro rings

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ABSTRACT

This study focuses on the expansion of metallic micro rings under internal pressure using a grain level three dimensional finite element analysis in combination with a rate dependent crystal plasticity constitutive model. The size effect due to grain statistics such as grain orientations and number of grains through the thickness is investigated to a certain depth, in terms of final geometries and the scatter involved. The findings of the study could be used as the preliminary guidelines for suitable material selection in stretching dominated microforming operations.



## 1. Introduction

The emergence of advanced technologies in many fields such as medical devices and micro-electronics has produced a significant demand for miniaturized metallic components. Forming technologies at conventional engineering scale are well established after many years of research and practical experience. Therefore, exploring this inheritance in the context of micro-forming would be the most rational way to develop cost effective, versatile micro-forming technologies. However, it has turned out that this route is not realizable due to the phenomena known as size effect [1].

There are different sources of size effect observed both at bulk response as well as frictional interaction with forming tools, as the size of the component approaches to the characteristics micro-structural size, e.g. average grain size [1,2]. The size effects associated with the bulk are typically classified as 'physical size effects' and 'statistical size effects'. The physical size effects are attributed to underlying dislocation mechanisms such as dislocation–grain boundary interaction and non-homogeneous dislocation evolution due to non-homogeneous plastic strains, which in turn influences the hardening behavior of the component as it is miniaturized, see e.g. [3,4].

As far as statistical size effect is concerned, a closer look at microstructure of a crystalline metal is essential where an assembly of grain is seen at the meso-level. Each grain has a lattice orientation which is one of the primary characteristics dictating the mechanical response upon external loading. Since the number of grains is diminishing as the component gets smaller, the behavior of individual grains becomes more pronounced and decisive on the overall component behavior. This fact leads to the 'statistical' size effect which is reflected in experiments as large scatters and dissimilar physical response [5].

Forming of very thin metallic micro components such as micro sheets, tubes and rings constitutes a non-negligible fraction of micro-component fabrication. Metallic micro-tubular products have typically very small thicknesses as compared to average grain size which implies that there exist relatively few number of grains through the thickness of such components. The forming and failure characteristics of these components differ significantly than that of macro tubular components, where failure loads and locations are predictable. On the contrary, failure in micro-tubes takes place randomly as experimentally demonstrated [6]. Number of grains and the adjacent grain orientations have a significant influence on the overall behavior and observed failure of micro-components.

As far as predictive modeling of these failures and micro-forming operations is concerned, conventional continuum plasticity theories are of almost no use. Crystal plasticity based constitutive description is essential in order to capture the deformation mechanisms of individual grains [6–11]. The good predictive capabilities of crystal plasticity approach are indeed verified by combined experimental–numerical studies [12,13]. It is important to note that the predictive capabilities of grain level crystal plasticity based models are closely related with the orientation measurements of the initial specimen/work piece including the inner grains, which is not a straight forward task.

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The mechanics of micro tube hydroforming [6], spring back of ultra thin sheet channels [8] have been studied using an explicit crystal plasticity finite element (CPFE) framework accompanied by Voronoi based microstructure generation tools [11] in a two-dimensional context. These studies provided insight in forming operations at small scales particularly investigating the influence of grain statistics on the measurable macroscopic quantities. However, it is to be noted that the grain level mechanical response is inherently anisotropic and requires a three-dimensional consideration due to random orientation distribution within the sample/workpiece. It seems that CPFE based 3D modeling of micro-forming operations is missing in the literature, which might be due to computational difficulties and cost as well as orientation characterization difficulties to be in turn used in the modeling phase.

Motivated by the lack of three-dimensional CPFE investigations in the literature, in this work, expansion of 3D micro rings under internal pressure is investigated using a crystal plasticity model within an implicit solution framework. Particularly, the influence of number of grains through the thickness in combination with random orientation distribution is investigated to a certain depth. Variation in deformed configurations in terms of scatter in final radii and thickness is systematically investigated. The findings of this study could be useful in the selection of suitable material microstructure for stretching dominated micro-forming operations.

The paper is organized as follows. In the next section, the crystal plasticity model is presented to a certain extent both in continuum and discrete forms including necessary algorithmic components that shall be used in finite element implementation phase. Afterwards, the quarter ring expansion analyses with microstructural variations are carried out systematically and the results are presented in appropriate forms. The following section is reserved for the analysis of these results in connection with the statistical size effects. In the last section, the findings of the study are summarized and potential extensions and improvements are highlighted.

## 2. Crystal plasticity model

In crystalline metals, inelastic deformation takes place by means of dislocation motion over well defined slip systems. In a continuum mechanics framework, these unit plasticity mechanisms are lumped in macroscopic deformation measures. In a geometrically nonlinear setting, the local deformation of a material point is determined by the deformation gradient tensor $\mathbf{F}$. This tensor is multiplicatively decomposed into an elastic part $\mathbf{F}_e$ and a plastic part $\mathbf{F}_p$, see Fig. 1, according to

$$\mathbf{F}=\mathbf{F}_e\mathbf{F}_p. \tag{1}$$

Plastic part of the deformation gradient maps the referential (undeformed) material line element to the intermediate stress free configuration as shown in Fig. 1. It is assumed that in the intermediate configuration, the orientation of the crystal lattice is identical to the orientation in the reference state. Therefore the intermediate configuration results from plastic shearing along well defined slip planes without changing the slip system orientations, see Fig. 1. The slip system labeled by $\alpha$ is described by slip plane unit normal vector $\mathbf{n}_0^\alpha$ and the orthogonal slip direction unit vector $\mathbf{s}_0^\alpha$ in the reference configuration. These statements imply that

$$\mathbf{n}^\alpha=\mathbf{F}_e^{-T}\mathbf{n}_0^\alpha \quad \text{and} \quad \mathbf{s}^\alpha=\mathbf{F}_e\mathbf{s}_0^\alpha \tag{2}$$

Using Eq. (1), the spatial velocity gradient can be expressed as

$$\mathbf{l}=\mathbf{l}_e+\mathbf{F}_e\mathbf{l}_p\mathbf{F}_e^{-1} \tag{3}$$

where $\mathbf{l}_e=\dot{\mathbf{F}}_e\mathbf{F}_e^{-1}$ and $\mathbf{l}_p=\dot{\mathbf{F}}_p\mathbf{F}_p^{-1}$. In crystal plasticity, $\mathbf{l}_p$ is supposed to be a summation of individual contributions of slip systems written as

$$\mathbf{l}_p=\sum_{\alpha=1}^{n_s}\dot{\gamma}^\alpha\mathbf{s}_0^\alpha\mathbf{n}_0^\alpha \tag{4}$$

where $\dot{\gamma}^\alpha$ is slip rate and $n_s$ is the number of active slip systems. It is to be noted that the current work focuses on the inelastic deformation of face centered cubic (fcc) metals which has 12 slip systems meaning that the opposite slip directions are not taken as distinct slip systems. Therefore $\mathbf{l}$ can be decomposed as

$$\mathbf{l}=\mathbf{l}_e+\sum_{\alpha=1}^{n_s}\dot{\gamma}^\alpha\mathbf{s}^\alpha\mathbf{n}^\alpha \tag{5}$$

by which $\dot{\gamma}^\alpha$ can be interpreted as slip rates with respect to the current slip orientation vectors; a consequence of Eq. (2).

Motivated by the fact that elastic strains are very small as compared to plastic strains in metal plasticity, St Venant's Kirchhoff material model, stating a linear relationship between the second Piola–Kirchhoff stress tensor $\mathbf{S}$ and elastic Green–Lagrange strain tensor $\mathbf{E}_e$ as

$$\mathbf{S}=\mathbb{C}:\mathbf{E}_e \quad \text{with } \mathbf{E}_e=\tfrac{1}{2}(\mathbf{F}_e^T\mathbf{F}_e-\mathbf{I}) \tag{6}$$

is used. Isotropic, fourth order elasticity tensor $\mathbb{C}$ is defined as

$$\mathbb{C}=\kappa\mathbf{I}\otimes\mathbf{I}+2G(\mathbb{I}-\tfrac{1}{3}\mathbf{I}\otimes\mathbf{I}) \tag{7}$$

with the bulk modulus $\kappa$, the shear modulus G and $\mathbb{I}$ is the fourth order symmetric identity tensor. The driving force for slip on system $\alpha$ is the Schmid stress $\tau^\alpha$ defined in the current configuration as

$$\tau^\alpha=\mathbf{s}^\alpha\boldsymbol{\tau}\mathbf{n}^\alpha \quad \text{with } \boldsymbol{\tau}=\mathbf{F}_e\mathbf{S}\mathbf{F}_e^T \tag{8}$$

where $\boldsymbol{\tau}$ is the Kirchhoff stress tensor. $\tau^\alpha$ is the conjugate of $\dot{\gamma}^\alpha$ in the sense that $\tau^\alpha\dot{\gamma}^\alpha$ is the contribution of the slip system $\alpha$ to the rate of plastic work. To complete the constitutive description, a relation between the conjugate pair $\tau^\alpha$ and $\dot{\gamma}^\alpha$ has to be defined. In this work, a power type description is adopted to determine the slip rate $\dot{\gamma}^\alpha$ on slip system $\alpha$, which reads as

$$\dot{\gamma}^\alpha=\dot{\gamma}_0^\alpha\frac{\tau^\alpha}{\tau_c^\alpha}\left(\frac{|\tau^\alpha|}{\tau_c^\alpha}\right)^{m-1} \quad \text{for } \alpha=1,2,\ldots,n_s \tag{9}$$

with material parameters $\dot{\gamma}_0^\alpha$ and $m$. $\tau_c^\alpha$ is representing the actual resistance on slip system $\alpha$ and evolves according to the following hardening rule:

$$\dot{\tau}_c^\alpha=\sum_{\beta=1}^{n_s}h^{\alpha\beta}|\dot{\gamma}^\beta| \text{ for } \alpha=1,2,\ldots,n_s \tag{10}$$

The particular form of the hardening moduli reads as

$$h^{\alpha\beta}=h^\beta(q+(1-q)\delta^{\alpha\beta}) \tag{11}$$

where $h^\beta$ is the self-hardening modulus of the slip system $\beta$ and $q$ is the latent hardening ratio.

Since an incremental formulation is necessary for the solution of the problem numerically, a time increment $t_n\le t\le t_{n+1}$ is considered. In a displacement driven context, an estimate for $\mathbf{F}_{n+1}$ is available. Furthermore the history variables $\mathbf{F}_{p_n}$, $\gamma_n^\alpha$, $\boldsymbol{\tau}_{c_n}$ are known. Relying on the time step sizes used in implicit FE solution procedure, it is assumed that the slip rates on individual slip systems are constant during the increment. Therefore one can write

$$\dot{\mathbf{F}}_p\mathbf{F}_p^{-1}=\frac{1}{\Delta t}\sum_{\alpha=1}^{n_s}\Delta\gamma^\alpha\mathbf{s}_0^\alpha\otimes\mathbf{n}_0^\alpha \tag{12}$$

The solution of the tensor ordinary differential equation (12) is given by

$$\mathbf{F}_{p_{n+1}}=\exp(\textstyle\sum\Delta\gamma^\alpha\mathbf{n}_0^\alpha\otimes\mathbf{s}_0^\alpha)\mathbf{F}_{p_n} \quad \text{with } \mathbf{F}_p|_{t=0}=\mathbf{I} \tag{13}$$

which requires the evaluation of the tensor exponential. This can