International Journal of Mechanical Sciences 87 (2014) 118–136

Contents lists available at ScienceDirect

International Journal of Mechanical Sciences

journal homepage: www.elsevier.com/locate/ijmecsci

Dynamic behavior analysis of a magnetorheological elastomer sandwich plate

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ARTICLE INFO

Article history:
Received 5 November 2013
Received in revised form
14 May 2014
Accepted 15 May 2014
Available online 4 June 2014

Keywords:
Magnetorheological structure
Vibration response
Shear modulus
Loss factor
Rigidity

ABSTRACT

The design of magnetorheological materials with mechanical properties adjusted to the action of dynamic loads is a recent field of research. The few bibliographic literatures in this area concern the beams. This paper is devoted to a numerical and an experimental study of the dynamic behavior of sandwich plates consisting of two aluminum skins and a magnetorheological elastomer (MRE) core of different loads of micron-size ferromagnetic particles elaborated under the action of a magnetic field.

Firstly, the rheological properties of the loaded elastomer with and without the impact of the magnetic field have been evaluated experimentally. Secondly, an experimental analysis of the impact of the loading rate of micron-size ferromagnetic particles of the elastomer as well as the magnetic field intensity on the vibration behavior of the elaborated plates is conducted. To evaluate the variation of the plate rigidity and damping factor, a confrontation of experimental values against numerical results, using the finite elements software Abaqus and the Ritz method of approximation for an appropriate model, was made for various dimensions and boundary conditions of the plate.

1. Introduction

Elastomers are a major asset in various industrial applications. Their specific mechanical properties, in terms of adapted hyper-elasticity give them excellent vibration and acoustic damping properties.

During these last years, research work has focused on the study of the magnetorheological properties of elastomers. Coquelle and Bossis [1] presented an experimental and a numerical method to determine the mechanical properties of elastomers controlled by a magnetic field. These elastomers are loaded using ferromagnetic particles, structured in unidirectional chains in a silicone elastomer matrix. Tests under the impact of a magnetic field showed a large increase of the mechanical properties. Hua-xia and Xing-long [2] based their study on the development of new parameters to identify the vibration behavior of magnetorheological elastomers, such as the depreciation rate of the viscoelastic properties. Jolly et al. [3], Demchuk and Kusmin [4] identified experimentally the impact of the magnetic field on the shear modulus of the MRE by evaluating the vibration amplitude of a magnetorheological elastomer sandwich beam. Davis [5] showed that the maximum variation of the shear modulus reached practically 50% of an elastomer loaded with 27% of ferromagnetic particles. The latter study has been verified experimentally by Zhou [6]. More recently, Zhou and Wang [7,8] conducted an analytical model of the vibrating motion of a sandwich beam under a uniform magnetic field, perpendicular to the direction of the thickness. To validate the analytical model, they conducted a second study based on numerical modeling to control the rheological properties. Dwivedy et al. [9] studied the parametric instability areas of a sandwich beam with a magnetorheological elastomer core identical to that studied by Zhou and Wang [7,8]; but subject to a periodic load, to determine the adjustment advantages of the mechanical properties of the loaded elastomers exposed to a magnetic field. They presented the method of Hsu [9] to determine areas of instabilities associated with the single and the combined resonance frequencies for different boundary conditions. More recently, Nayak et al. [10,11] have studied the vibration reduction of a sandwich beam with three cores, an unloaded elastomer core, a loaded elastomer core with ferromagnetic particles and a loaded core with particles of black carbon. Areas of parametric instability for the two different loading types of particles have been studied only for the first three vibration modes to illustrate the passive and the active vibration reduction. In this study, Boczkowska et al. [12] studied a magnetorheological elastomer (MRE) consisting of carbonyl-iron particles in a polyurethane matrix. The storage modulus was measured as a function of

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http://dx.doi.org/10.1016/j.ijmecsci.2014.05.014
0020-7403/

angular frequency with or without the strength of magnetic field. Han et al. [13] studied both the shear and the axial deformation of the material, with a magnetic field applied on the particle chain direction. It is found that the magnetic interaction between particles is indeed the major cause of the stiffening effect. Ying et al. [14] studied stochastic excitation forces of a magnetorheological sandwich plate simply supported on its all four sides by the approximate Galerkin's method. Yeh [15] studied the case of free vibrations of a magnetorheological sandwich plate simply supported on all its four sides by the finite element method. He determined the natural frequency and the modal loss factor versus the Mode number of the plates and the magnetic field, for the easiest case when the relations of approximation are known. Xiao-min et al. [16] have studied numerically a new variable stiffness absorber based on magnetorheological elastomer. He considered the acoustic emission coefficients, the storage modulus, and the plate dimensions according to the frequencies. Miao et al. [17] have studied the mechanical and the dynamic variations properties of an elastomer loaded with carbon and iron particles, by varying the intensity of the magnetic field. Magnetorheological structures are a new area where almost all of the research has been focused on the beams. Extending these investigations to other configurations of elements such as plates, aircraft blades, aerospace or other applications and taking into account the non-linear dynamic loads would lead to promising results.

In this paper, an experimental and a numerical study of the vibration response of magnetorheological sandwich plates, subjected to a varying magnetic field under a forced excitation is presented. First, an experimental study of the rheological properties of an elastomer loaded at 30% of ferromagnetic particles of micrometric size is conducted. This is subjected to a magnetic field as to evaluate the impact of the magnetic field.

Second, three sandwiched plates, whose cores of elastomer loaded with different percentages of ferromagnetic particles, are prepared. They are subject to a constant magnetic field until the complete cross linking chains of their elastomers. An experimental analysis in a variable magnetic field with a forced excitation was conducted.

An approximate formulation by Ritz method for modeling the vibration behavior of plates is developed and the results are simulated using Matlab software. Finally, in order to validate the results on the characteristics of vibration behavior in a magnetic field of the plates found by the two previous methods, a finite elements model of the Abaqus software was developed with an estimation of geometry and the boundary conditions of embedding plates. A procedure for minimizing errors between the data extracted by both numerical approaches is adapted for each iteration of the simulation. A comparative study between the three methods is performed.

2. Mathematical formulation of the problem

Let us consider a rectangular sandwich plate of dimensions $a \times b$, thickness h and mass ρ_s per unit area. The boundary conditions of this plate are embedded free, Fig. 1(a). This plate is subjected to a magnetic force F_m, Fig. 1(b).where d_i ($i=t,b,c$) are the thickness skins and MRE core.

The principle of Hamilton and Ritz method are used to determine the equations of motion of the sandwich plate.

Hamilton's principle of a mechanical system [18,19] from instant t_1 to instant t_2 is given by

$$\int_{t_1}^{t_2} \delta(T-U)dt + \int_{t_1}^{t_2} \delta W\, dt = 0 \tag{1}$$

where T is the kinetic energy, U the strain energy, W the work of applied forces, δ the variational operator, and t_1 and t_2 the time variations.

The horizontal displacements of sandwich plate are respectively equal to $u=v=0$. The ordinary differential equation of motion in bending is written as [14,20]

$$M\frac{d^2w}{dt^2} + Kw = F \tag{2}$$

where M and K are respectively the global mass matrix and the global rigidity matrix, which are defined as

$$M = \begin{bmatrix} M_{11} & . & . & . & M_{1i} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ M_{m1} & . & . & . & M_{ij} \end{bmatrix}, \quad K = \begin{bmatrix} K_{11} & . & . & . & K_{1i} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ K_{m1} & . & . & . & K_{ij} \end{bmatrix} \tag{3}$$

Eq. (2) represents a magnetic excited multi-degree-of-freedom system derived from an MRE sandwich plate, which has a complex rigidity K dependent on the vibration frequency and controllable by applied magnetic fields due to G_c^*, determined experimentally (Table 3, Appendix B) and represented after mathematical development by Eq. (33) [14]. The frequency response function matrix of the system (2) is expressed by Eqs. (20), [15,21]. The same equation has been given by Bilasse [21] in the forced vibration case and by Yeh [15] for free vibration. The second member of Eq. (2) is

$$F = \alpha + F_m$$

where α is the distributed equivalent horizontal force and the moments induced by the magnetoelastic load on the top (t) and bottom (b) skins respectively (see Fig. 15, coil 6). These terms of the magnetoelastic loads represented in [8–10] will be more explicit by Eqs. (15) and (16); F_m is the excitation magnetic force generated by coils 4 (see Fig. 15a) and magnet (Fig. 15b). The generalized excitation vector of magnetic field is detailed in Eq. (35).

The general analytical solution of the homogeneous differential Eq. (2) is expressed as [22]

$$w(x,y,t) = w(x,y)e^{i\lambda t} \tag{4}$$

In this study, the forced vibrations of plate are considered. The kinetic and strain energies of each part of the plate (skins and MRE core) and the generated work by the magnetic field are more detailed as follows. Zhou and Wang [8], and Dwivedy et al. [9] represented, for the sandwich beam, the first variation of the kinetic energy and that of the strain energy without the effect of the magnetic field (as of energy in a conventional beam); and the work by external forces expressed by the moments and the forces created by the magnetic field effect.

Furthermore, the equations of strain and kinetic energies of MRE plate are presented more explicitly by Yeh [15]; where the shear complex modulus of MRE is obtained and controlled experimentally under the effect of magnetic field and more explained in the experimental tests section.

In our case, the total kinetic energy of the sandwich plate is given by [15,19]

$$T = T_t + T_b + T_C \tag{5}$$

2.1. Kinetic energy of sandwich plate

Kinetic energy of the top skin is

$$T_t = \frac{1}{2}\int_0^a \int_0^b \left[\rho_t d_t^2 \left(\frac{\partial w_t}{\partial t}\right)^2 + \rho_t d_t^3 \left(\left(\frac{\partial \dot{w}_t}{\partial x}\right)^2 + \left(\frac{\partial \dot{w}_t}{\partial y}\right)^2 \right) \right] dx_t\, dy_t \tag{6}$$