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Analytical model for tube hydro-bulging test, part I: Models for stress components and bulging zone profile

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ABSTRACT

Hydro-bulging test is an advanced method for characterizing the mechanical properties of tubular materials under bi-axial stress state, by measuring the bulging height, pole thickness and internal pressure. This method has been investigated and applied over the past decades. However, different testing conditions and analytical models have been used in practice, which makes the tested results incomparable. In this paper, hydro-bulging tests with different constraining are first analyzed and compared, and a unified analytical model for stress components calculation is proposed for closed-ends and fixed-ends conditions. Finite element analysis of tube hydro-bulging with closed-ends and fixed-ends shows that both the stress and strain responses of the tubular specimen, i.e., the stress and strain components under different internal pressures, are almost identical to determine the stress–strain relations. Geometrical models for bulging zone are analyzed and compared with special attention on the calculation of the radius of axial curvature, which is one of the key parameters for both stress and strain components determination. A die-related ellipsoid model is presented and is well verified by experimental results on extruded AA6061 tubes.

1. Introduction

Tube hydroforming has achieved wide applications in automotive industry over the past decades, especially for tubular steel materials. Facing an increasing environmental demands, lightweight materials such as aluminum alloy and magnesium alloy have become ideal alternative materials for traditional steels [1,2].

As steel tubes are generally roll-formed from sheet, it is reasonable to characterize the properties of tubular material using the properties of sheet metal. In practice, uniaxial tensile test is often adopted to test the mechanical properties of raw sheets along different directions. For tubular material, the properties along axial direction can be tested directly by uniaxial tensile test, too. However, extruded tubes are highly anisotropic compared to roll-formed steel tubes. Both the ductility and the mechanical properties along axial direction and hoop direction are generally quite different [3–7]. Therefore, accurate mechanical properties along other directions are thus required for modeling the tubular material. In order to avoid the effect of specimen preparation for uniaxial tensile test, so-called ring hoop tension test (RHTT) was developed to determine the properties of tubes along circumferential direction [8–10]. Unfortunately, the strength of the specimen is often overestimated due to the friction force between D-blocks and specimen [11].

In order to provide more accurate mechanical properties of tubular materials resulted from different forming processes, direct testing methods should be adopted during which similar deformation model or stress states are followed [12,13]. In a hydroforming process, the material deforms predominantly in a biaxial stress state. Therefore, hydro-bulging test has become one reasonable and promising choice. Different testing methods and apparatus have been developed and applied, and measurement of biaxial stress–strain curves of tubular materials or sheet metals under biaxial tension can be realized [14–22]. In all these tests, four types of end-conditions, i.e., free-end [16,21], closed-end [14], fixed-end [15,19,20,22] and forced-end [17,18], are often used. In all these tests, hydro-bulging with fixed-end is more appropriate for determining the flow behavior of tubes, because it is relatively easy to realize. At the same time, the biaxial stress-state is similar to the typical hydroforming process and the deformation paths are simple, which makes the tested results stable and comparable, just like the traditional uniaxial tensile test. However, no explicit formulation for stress components can be given directly; approximations are often followed [17–20,22]. The validity of this approximation and its effect on consequent results have not been fully discussed up to now.

Curvature radius along hoop direction and axial direction in the bulging zone is also required for stress and strain components

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calculation. The profile of bulging zone can be determined according to the bulging displacement measured by displacement transducers, in conjunction with an assumption of profile curve of the bulging zone [14,17–20,22], or measured directly according to the real shape of a tested tube [23,24]. The assumption of profile curve of the bulging-zone can be classified into two groups, i.e., considering [19] or neglecting [22,25,26] the die entry radius, which will definitely affect the accuracy for describing the profile of the bulging zone and consequently the final stress–strain curve [27].

In this study, the relation between tests with different end conditions will be discussed first, by comparing the data required for calculating equivalent stress–strain curve. A geometrical model of describing the profile of bulging zone is also analyzed by numerical simulation and verified by experiments. The change of pole thickness during hydro-bulging, which is also a key factor for hydro-bulging test of tubular materials, will be discussed in an accompanying paper, i.e. Part II [28].

2. Analytical model for stress

2.1. Hydro-bulging test: principle

As mentioned above, hydro-bulging test is an advanced method for characterizing the deformation behavior of tubular materials. In a hydro-bulging process, tubular specimen is expanded under biaxial stress state, and an equivalent stress–strain curve can be obtained directly from this test. Such obtained equivalent stress–strain relations can well describe the properties of tubular materials for industrial application, such as hydroforming processes. Fig. 1 shows a schematic diagram of tube hydro-bulging test. Tubular specimen is freely expanded by internal pressure, with certain length of bulging zone L_0 and end-constraints. During hydro-bulging, the bulging height h, pole thickness t_P and shape of central bulging zone under given internal pressure p are recorded, and then all the stress and strain components can be calculated to determine the equivalent stress–strain curve.

Strain components in middle plane at point P along hoop and radial directions can be given as

$$\varepsilon_\theta = \ln\left(\frac{R_P - t_P/2}{R_0 - t_0/2}\right) = \ln\left(\frac{h + R_0 - t_P/2}{R_0 - t_0/2}\right) \tag{1}$$

$$\varepsilon_t = \ln\left(\frac{t_P}{t_0}\right) \tag{2}$$

where R_P is the outer radius of the deformed tube at point P.

By considering volume constancy $\varepsilon_\theta + \varepsilon_t + \varepsilon_z = 0$, all the strain components can be calculated if bulging height h and pole thickness t_P are measured during the hydro-bulging process.

Fig. 2 shows the stress analysis of the central point P in the bulging zone. In the figure, ρ_θ and ρ_z, and σ_θ and σ_z are curvature radii and stresses in the hoop and axial directions, respectively.

For a thin-walled tube, the stress in the thickness direction can be neglected, and the tube is loaded under plane-stress state. Only two stress components along the hoop direction and axial direction are required to calculate the equivalent stress. Analytical models for stress components will be discussed in Section 2.2, because they depend on the end-constraints of tubular specimen during the hydro-bulging process. According to the equilibrium of the shell element shown in Fig. 2, the following relation holds:

$$\begin{aligned} &2\sigma_z \cdot (dl \cdot t_P) \cdot \ \sin\left(\frac{1}{2}dz\right) + 2\sigma_\theta \cdot (dm \cdot t_P) \cdot \ \sin\left(\frac{1}{2}d\theta\right) \\ &\quad = p(\rho_z - t_P) \cdot \ \sin\, dz \cdot (\rho_\theta - t_P) \cdot \ \sin\, d\theta \end{aligned} \tag{3}$$

For small element, $\sin((1/2)(dz)) = (1/2)dz$, $\sin((1/2)(d\theta)) = (1/2)d\theta$, $\sin(dz) = dz$ and $\sin(d\theta) = d\theta$. Angles dz and $d\theta$ can be written as

$$\begin{cases} dz = \dfrac{dm}{\rho_z - t_P/2} \\ d\theta = \dfrac{dl}{\rho_\theta - t_P/2} \end{cases} \tag{4}$$

Therefore, Eq. (3) can be re-written as

$$\sigma_z \cdot \frac{t_P}{(\rho_z - t_P/2)} + \sigma_\theta \cdot \frac{t_P}{(\rho_\theta - t_P/2)} = p\frac{(\rho_z - t_P)(\rho_\theta - t_P)}{(\rho_z - t_P/2)(\rho_\theta - t_P/2)} \tag{5}$$

For extruded tubular materials with transverse anisotropy only, and ignoring the elastic deformation, then the Hosford yield criterion based on crystal plasticity for plane stress problem can

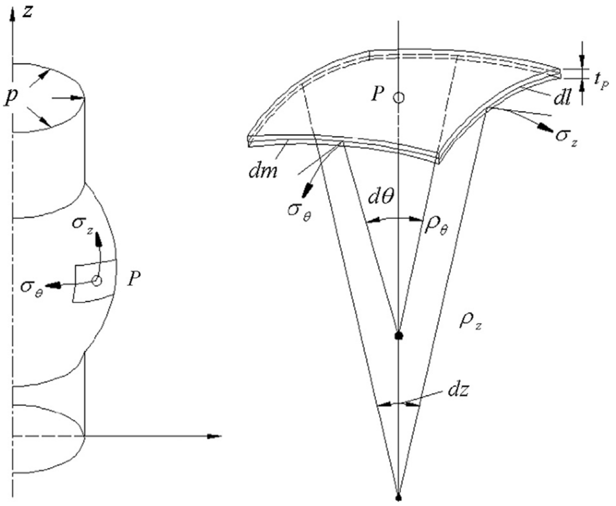

Fig. 2. Stress analysis at the central position of bulging zone.

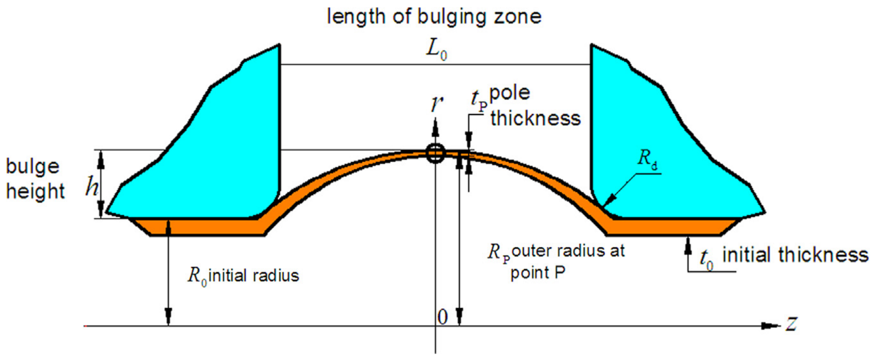

Fig. 1. Schematic diagram of tube hydro-bulging test.