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Bifurcation analysis of nonlinear milling process with tool wear and process damping: Sub-harmonic resonance under regenerative chatter

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ABSTRACT

In this paper, bifurcation analysis is performed for the nonlinear milling process under sub-harmonic resonance and regenerative chatter, with tool wear and process damping effects. Multiple-scales approach is used to construct analytical approximate solutions for non-autonomous parametrically excited equations of the system with time delay terms. The new bifurcation parameters are the detuning parameter (deviation of the tooth passing frequency from three times of the chatter frequency), damping ratio (affected by process damping) and tool wear width. Jump phenomenon and multi-values responses are observed in the first order solution under sub-harmonic resonance condition. Periodic, quasi-periodic and chaotic behaviour of the limit cycles are predicted in the presence of regenerative chatter. Change of the detuning parameter leads to the cyclic-fold (tangent) and secondary Hopf (Neimark) bifurcations. It is observed that as damping (affected by process damping) varies, cyclic-fold, secondary Hopf and supercritical symmetry-breaking (pitchfork) bifurcations occur. For the variation of tool wear width, period-doubling (flip), symmetry-breaking and secondary Hopf bifurcations occur. Moreover, for slight values of damping or large values of tool wear width, chaotic behaviour is dominant.

1. Introduction

Peripheral milling is extensively used in manufacturing processes, especially in aerospace industry where end mills are used for milling of wing parts and engine components. Generation of complex shapes with high quality for various types of materials is the main advantage of milling in contrast to other machining processes. However, the occurrence of self-excited vibration or chatter may cause damage to tool, spindle bearing or the work-piece, and may result in poor dimensional accuracy and surface finish of the work-piece.

Regeneration and mode coupling are two well known mechanisms that cause machine tool chatter [1]. In most machining processes, the regenerative type is found to be the main mode of chatter. It occurs when the oscillation of the depth of cut in one pass of the tool leaves waves on the machined surface that are regenerated in the subsequent passes of cut. Under such conditions, the dynamics of the cutting system is defined through a coupled set of delayed differential equations [1]. For cutting force modelling in various milling operations and developing the dynamic equations of the process to predict the deflection of machine components and to estimate form errors, mechanistic approach has been used extensively [2]. In this approach, cutting force coefficients are calibrated for certain cutting conditions using experimental data [3,4]. Once the flank forces, friction and shear angles are determined from orthogonal cutting tests, the milling forces are calculated by transforming the orthogonal conditions to oblique cutting conditions [5].

For the purpose of chatter prediction and the conditions under which such instability occurs in the milling process, the recent trend toward high speed and high precision cutting processes requires more accurate modelling of the process. Frictional conditions at the tool-workpiece interface affect the dynamic cutting force characteristics and consequently the process stability and production rate. In an early work, the flank wear model for milling process was presented considering the abrasion and diffusion mechanisms, thermal fatigue effect and number of teeth [6]. Also, tool flank wear effects have been modeled through an analytic mechanistic model of milling process [7].

At low cutting speeds, process damping (arisen from cutter-workpiece interaction) has a significant role on increasing stability in machining. Identification of process damping from tool vibration measurements [8], mechanistic modelling of milling with process damping [9,10], and identification of cutting force coefficients and chatter stability with process damping [11] are some of the recent works in this area. More recently, a practical

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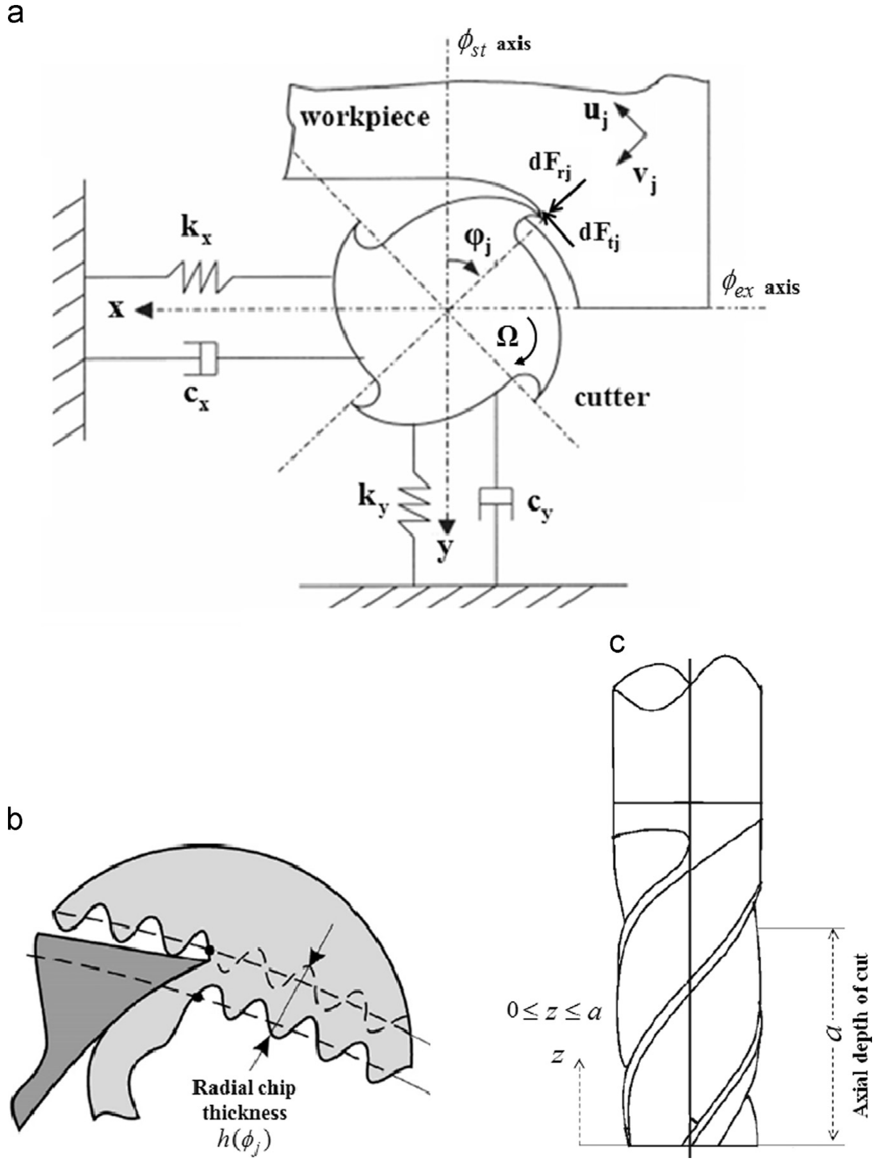

Fig. 1. (a) Dynamics of the milling process, (b) radial chip thickness in regenerative chatter (Source of part (b): [27]) and (c) axial depth of cut.

identification and modelling of process damping coefficients based on chatter tests has been presented that determines the indentation force coefficient responsible for the process damping [12].

Nonlinear modelling of the chatter phenomenon has received much attention in recent years, because the stability theories based on linear dynamics cannot predict some of the interesting phenomena in milling process. Delayed nonlinear models with square and cubic polynomial terms related to the cutting force and structural stiffness were considered [13,14]. Also, nonlinear analysis of regenerative chatter using a power-law function for cutting force was carried out [15,16]. For nonlinear dynamic analysis of the problem, several techniques such as center manifold theory, bifurcation theory, perturbation analysis, phase portraits and Poincare sections have been used. The existence of sub-critical Hopf and period doubling (flip) bifurcations in the delay differential equation for regenerative chatter has been shown using the center manifold theory and Poincare sections [17,18]. Bifurcations and limit cycle behaviour in milling process are predicted from a non-linear time finite element analysis (TFEA) [19]. In other works, period doubling and Hopf bifurcation in low-immersion high speed machining and bifurcations caused by structural asymmetry have been studied [20].

Although major sources of nonlinearity have been identified and complex models show appropriate approximations to the physical phenomenon [21], there are still room for more comprehensive models of the process that include more complexities of the milling process. In the previous researches [22–24], the extended nonlinear modelling of cutting forces and development of closed form expression through Fourier series expansion have been accomplished. Considering the structural and cutting forces nonlinearities, dynamics of regenerative chatter and internal resonance phenomenon was investigated [22]. In the other research, nonlinear dynamics and bifurcation analysis of the milling process in the presence of tool wear, process damping and cutting force nonlinearities was studied [23], while in [24], a tunable vibration absorber was designed to suppress regenerative chatter and improve stability limits of the process.

In this paper, the extended dynamic model of the peripheral milling process with tool flank wear, process damping and nonlinearities in regenerative chatter terms is considered. After determination of the closed form expressions for the nonlinear cutting forces, non-autonomous parametrically excited equations of the system with time delay terms are developed. Unlike the previous research dealing with the primary resonance case [23],