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Optimum design of laminates containing an elliptical hole

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ABSTRACT

An optimum design of a 4, 8 and 16 layered graphite/epoxy and glass/epoxy symmetric laminated plate, containing an elliptical hole and subjected to various in-plane loading conditions is presented using Muskhelishvili's complex variable approach and hybrid genetic algorithm (GA). The Tsai–Hill criterion and quadratic interaction failure criterion are taken as an objective function for single plate lamina and symmetric laminate respectively. The ply orientation angles for different lamina are considered design variables. In the genetic algorithm (GA), tournament selection and heuristic crossover are used. The elitist model is also applied for the effective reproduction in the population.

1. Introduction

Light weight and superior strength compared to metals makes composite materials well chosen for a wide variety of applications like aircraft, space vehicles, automobiles, biomedical, etc. The mechanical and structural components in such applications require cutouts for satisfying certain service demands. These cutouts adversely affect the stress pattern of the mechanical/structural components. Hence it is essential to know the stress pattern around such holes.

The stress field around holes in composite structures depends upon fiber orientation, stacking sequence, loading angle and hole geometry [1–7]. The stress analysis around different shaped holes was carried out by Savin [1], Lekhnitskii [2], Ukadgaonker and Rao [3,4], Ukadgaonker and Kakhandki [5], Rao et al. [6], Ukadgaonker and Awasare [8–11], Rezaeepazhand and Jafari [12], Batista [13], Sharma [7,14] etc., considering various types of loading conditions, for isotropic as well as anisotropic plates, using Muskhelishvili's complex variable approach [15]. The unique characteristic of composite materials is that the fiber orientation and stacking sequence can be controlled to meet the required strength and stiffness for the specific choice. The complex behavior and a large number of design variables in composites require sound knowledge of various optimization techniques to achieve these properties. The optimization of composite plate containing a hole, under various types of loading conditions, is an interesting domain of investigation.

The genetic algorithm, a subset of an evolutionary algorithm, is a sparingly useful tool for optimization [16] of problems (with multiple local optimum and with more number of design parameters) because it is population based, probabilistic in nature, works on coded parameters, needs no auxiliary information, etc. Genetic algorithm has been a very popular method for optimizing the stacking sequence of a laminated composite [17] and the best tool to optimize composite laminate [18,19]. Callahan and Weeks [20], Le Riche and Haftka [21], Nagendra et al. [22], Ball et al. [16], Park et al. [23] and Lopez et al. [24] were the first few researchers who have adopted and used the genetic algorithm for optimization of stacking sequence in the laminated composite plate. Genetic algorithm has also been applied in conjunction with finite element analysis [21,23,25,26]. The approximate optimum solution of orientation and thickness ratio of laminate having elliptical opening has been presented by using an iterative method in conjunction with quadratic failure criterion by Tan [27].

The optimum design of a composite plate with a circular/elliptical hole under various loading conditions, and the fusion of complex variable approach and GA is not available in the existing literatures. Here, present work addresses the research in domain of optimization of composites by using GA in conjunction with complex variable approach. GA is hybridized with the pattern search method to find the best global solution that is applied at the end of optimization process.

The purpose of this paper is to obtain the best fiber angle in perforated orthotropic plate and stacking sequence in symmetric

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composite plate containing an elliptical hole when it is subjected to in-plane loading. The genetic algorithm is used to optimize the fiber angle/stacking sequence based upon the strength that is calculated by using Muskhelishvili's complex variable approach [15]. The Tsai–Hill criterion [28] and quadratic failure criterion [29,30] are used as an objective function and fiber orientation is used as a design variable. A 1 (single lamina), 4, 8, and 16 layered symmetric laminate of graphite/epoxy and glass/epoxy is considered throughout this study.

2. Mathematical formulation

A thin anisotropic infinite plate is considered under plane stress conditions. Each lamina is assumed homogeneous and of uniform thickness. Based on Muskhelishvili's complex variable approach [15], the bi-harmonic function $U(x,y)$ can be represented in terms of complex variable $z=x+iy$. The function can be written as

$$U=2\,\mathrm{Re}[\varphi(z_1)+\chi(z_2)], \tag{1}$$

$\varphi(z_1)$ and $\chi(z_2)$ are the functions of complex variable $z_i=x\pm\mu_i y$, $i=1,2.\mu_i$ $(i=1,2)$ are the constants of anisotropy, calculated from the characteristic equation. The material constituent relations, Airy's stress function and strain–displacement compatibility conditions are employed to obtain characteristic equation [2].

The stress components, bounded throughout the region, are to be given by

$$\sigma_x=2\,\mathrm{Re}[\mu_1^2\varphi'(z_1)+\mu_2^2\psi'(z_2)],$$

$$\sigma_y=2\,\mathrm{Re}[\varphi'(z_1)+\psi'(z_2)], \tag{2}$$

$$\tau_{xy}=-2\,\mathrm{Re}[\mu_1\phi'(z_1)+\mu_2\psi'(z_2)],$$

where $\phi(z_1)=d\varphi/dz_1$; $\psi(z_2)=d\chi/dz_2$, $\phi(z_1)$ and $\psi(z_2)$ are the functions of complex variable $z_i=x\pm\mu_i y$, $i=1,2$.

The area external to a given elliptical hole (Z-plane) is mapped conformally to the area outside the unit circle (ζ-plane) using the following mapping function:

$$z_j=\omega_j(\zeta)=\frac{R}{2}\left[a_j\left\{\frac{1}{\zeta}+\left(\frac{a-b}{a+b}\right)\zeta\right\}+b_j\left\{\zeta+\left(\frac{a-b}{a+b}\right)\frac{1}{\zeta}\right\}\right]\quad(j=1,2) \tag{3}$$

where $\zeta=\rho e^{i\beta}$, $a=$ semi major axis, $b=$ semi minor axis, $a_j=1+i\mu_j$, $b_j=1-i\mu_j$.

The plate is assumed to be loaded about arbitrary coordinate axis $X'-Y'$, which makes an angle α with X–Y axis in the principal direction (refer Fig. 1). The top and bottom surfaces of plate are free from external forces.

The stress functions for the plate loaded at infinity are to be written as [15]

$$\phi(z_1)=-\frac{X^*+iY^*}{2\pi(1+\kappa)}\log z_1+\phi_1(z_1)+\phi_0(z_1),$$

$$\psi(z_2)=\frac{\kappa(X^*+iY^*)}{2\pi(1+\kappa)}\log z_2+\psi_1(z_2)+\psi_0(z_2), \tag{4}$$

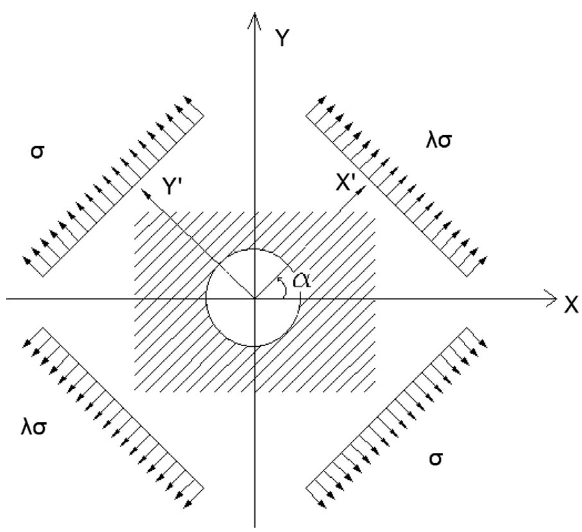

Fig. 1. Plate with a hole.

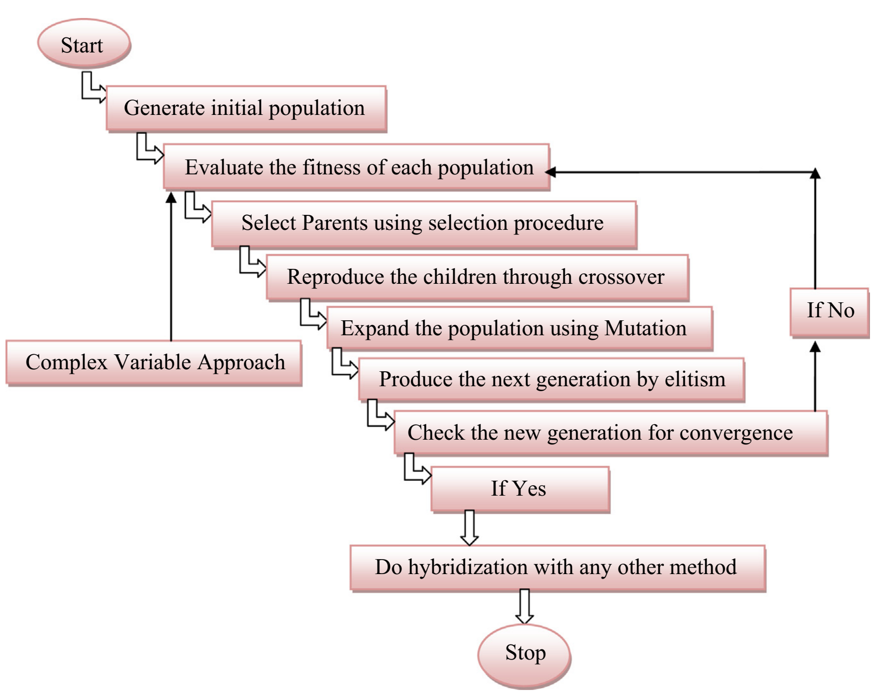

Fig. 2. Flow chart of genetic algorithm.